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Optimization of Stock Diversification with Quadratic Programming Method Using Lingo

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Abstract

Portofolio theory emphasizes the search for the optimal combination of investments that gives the maximum profit rate at a particular level of risk. Portfolios are formed by investors in general is a portfolio based on the investor preferences. Portfolios are formed based on the preferences of the portfolio is not really optimal. To determine the composition of shares that must be invested in order to generate optimal portfolios with constraints is by way of optimization techniques.

In investing, the investor's expectation is to obtain a return that is as large as possible or at least greater than the interest rate of deposits. In reducing investment risk, the better investment is done not only in one area. The expectation is if the loss of investment in a field then the loss is expected to be reduced or eliminated from investment in other areas (diversification). When diversifying, investors usually confuse in selecting and determining the proportion of investment he has to do (Optimal). Therefore, investors need to have a grip in diversification so that investor expectations can be fulfilled. One of them is by diversifying with quadratic programming method. Quadratic programming processes conducted based on Markowitz's model approach using LINGO. The results of this research in the form of investment proportions on some stocks. The research method used is a descriptive and verification method with techniques aimed at sampling such as sampling. The description of the calculations in this study uses secondary data with a sample of historical data shares included in the LQ45 index list during the period February 2004 to July 2009. The data is then processed using the quadratic programming method

The results showed that the optimal diversification by using the quadratic programming method will be achieved when the investment made in the shares of Indosat Tbk (ISAT) of 13,2%, Indofood Sukses Makmur Tbk (INDF) of 4,12%, Bank Central Asia Tbk(BBCA) of 37,55%, Astra International Tbk (ASII) of 4,11%, Aneka Tambang (Persero) Tbk (ANTM) of 0,91%, Astra Agro Lestari Tbk (AALI) of 5,73%, Telekomunikasi Indonesia Tbk (TLKM) of 31,9% and shares of Holcim Indonesia Tbk (SMCB) of 2,45%. Risks arising from the limitations of existing investments and amounted to 0,14%.

Keywords: portofolio optimization, Markowitz model, diversification, constraints, quadratic programming.

INTRODUCTION SECTION

Stocks have characteristics known as high risk - high return. On the one hand, investors as shareholders may be able to get large returns or profits in a short time. But on the other hand, along with fluctuating stock prices, the stock can also make investors experience large losses in a short time.

On the secondary market (stock exchange), investors gain return from the difference in the buying price and sell their stocks. In diversification, for some investors it will be easy to do, but for novice investors it is a difficult thing to do. Therefore, it takes a way that investors especially novice investors can diversify easily. Optimization can be carried out to reduce the risk which is caused by a diversification formed based on preferences, so that the resulting portfolio is an optimal portfolio based on historical data. This is carried out because the formed portfoliosbased on preferences are not truly optimal portfolios (Hartono, 2015).

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The optimization problem can be divided into two, namely linear and non linear. The role of linear programming in all fields is very important. The main assumption of linear programming is that all the functions involved (objective functions and constraint functions) are linear. Although basically applied to many practical problems, this assumption is often not appropriate (Hillier, Lieberman, Nag, & Basu, 2017). In the problem of the degree of nonlinearity, the optimization function used is the non-linear programming function. Alternative formulations to solve optimization problems that have these constraints can be solved using quadratic programming methods.The strength of the quadratic programming method is that it has a model advantage to quickly calculate a quantitative problem when the model has been formulated (Hillier, Lieberman, Nag, & Basu, 2017).In fact, many economists find a degree of nonlinearity in some cases.

The purpose of this research is to find out the investment composition of a number of shares that need to be owned by investors so that the optimization is expected to be achieved through the quadratic programming method using lingo.

LITERATURE REVIEW

In diversification when an investor owns a number of stocks it is better than owning one, but there is a condition where increasing the number of stocks makes a difference. There is a debate over how many stocks are needed to reduce risk while maintaining a high return (Investopedia, 2019). Research conducted by Lawrence Fisher and James H. Lorie states that to reduce the distribution of returns from the number of stocks in a portfolio requires 32 random stocks which can reduce distribution by 95%, compared to portfolio of the entire New York Stock Exchange (Fisher & Lorie, 1970).

This is felt to be inappropriate if we diversify enough to only 32 stocks. So it is necessary to look for other ways so that the return obtained is as expected. Many optimization methods that we can use in diversifying. One of them is quadratic programming (QP). Many say that this method is difficult and difficult to use so that many people use other methods in looking for portfolio optimization. This can be removed when we use the LINGO application.

Optimal portfolios can be obtained through the Markowitz portfolio model. The optimal portfolio has the following assumptions (Hartono, 2015).

a. The time used is only one period
b. There are no transaction charges
c. Investor preferences are only based on expected returns and risks of the portfolio
d. There are no loans and no risk deposits

Two things were considered by Markowitz when formulating a portfolio optimization model (Karacabey, 2007). First, investors will maximize expected return. Second, investors consider the expected returns and the variance of unexpected returns. In the Markowitz optimization model, variance is used to measure risk. The purpose of this model is to find weights that can minimize variance and ensure returns equal to or greater than expected returns.

The objective function in the Markowitz model will be minimized by several constraints. That is (Hartono, 2015):

a. The total proportion invested in each asset for all assets is equal to 1. If is the *i*-th proportion of assets which is invested in a portfolio consisting of n assets, then this first constraint is written as follows:

$$\sum_{i=1}^{n} w_i = 1$$

b. The proportion of each security must not be negative, so it has the following equation: $w_i \geq 0$ for i = 1 to*n*

c. The average amount of all returns for each asset (*Ri*) is equal to the portfolio return (*Rp*)

$$\sum_{i=1}^{n} w_i . R_i = R_P$$

The optimization model of quadratic programming method has linear constraints, but the objective function is quadratic. Although this method uses the same simplex table with Wolfe's algorithm in linear programming, but in quadratic programming Wolfe added artificial variables to the simplex table.

Problems in quadratic programming can be formulated as follows (Taha, 2017):

$$\text{Maximization} z = cx + x^TDx$$

Subject to

$$Ax \leq b \text{ , } x \geq 0$$

where

$x = (x_1, x_2, \dots , x_n)^T$

$c = (c_1, c_2, \dots , c_n)$

$b = (b_1, b_2, \dots , b_n)^T$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

$$D = \begin{pmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \vdots & \vdots \\ d_{m1} & \cdots & d_{mn} \end{pmatrix}$$

c is a row vector, x and b are column vectors, D and A are matrices, and the power of T indicates the transposition. The x^TDx function shows quadratic. The solution to the quadratic programming problem is based on the conditions of Karush Kuhn Tucker (KKT). So the above equation can be written as follows:

$$\text{Maximization} z = cx + x^TDx$$

subject to

$$G(X) = \begin{pmatrix} A \\ -I \end{pmatrix} X - \begin{pmatrix} b \\ 0 \end{pmatrix} \leq 0$$

where

$\lambda = \lambda_1, \lambda_2, \dots , \lambda_m)^T$

$U = (\mu_1, \mu_2, \dots \mu_n)^T$

Become a Lagrange multipliers of two constraints$AX - b \leq 0$ dan $-X \leq 0$. The application conditions of Karush Kuhn Tucker's are combined as follows

$$\begin{pmatrix} -2D & A^T & -I & 0 \\ A & 0 & 0 & I \end{pmatrix} \begin{pmatrix} X \\ \lambda \\ U \\ S \end{pmatrix} = \begin{pmatrix} C^T \\ b \end{pmatrix}$$

$$\mu_j x_j = 0 = \lambda_i S_i \text{ for all i and j}$$

$$\lambda, U, X, S \geq 0$$

where:

X :	measured variable
D :	matrix of quadratic variables on the objective function
C :	matrix of non quadratic variableon the objective function
b :	constraint value on the right hand side
I :	identity matrix
A :	matrix in the constraint
λ :	Lagrange multiplier variable
U :	Karush-Kuhn-Tucker variable
S :	slack variable

Then use phase 1 of the two phase method to find a basic feasible (BF) solution to the problem.

EQUATION AND MATHEMATICS

This optimization is obtained through data of secondary in the form of stock prices in the stock market. Based on these data, the covariance of each share is calculated.

The formula used to calculate covariance through historical data is as follows (Hartono, 2015):

$$\operatorname{cov}(R_A, R_B) = \sigma_{R_A, R_B} = \sum_{i=1}^{n} \frac{[R_{Ai} - E(R_A)].[R_{Bi} - E(R_B)]}{n}$$

where

$Cov(R_A,R_B)$	=	covariance of return between stock A and stock B
R_{Ai}	=	future return of stock A on the i-th condition.
R_{Bi}	=	future return of stock B on the i-th condition.
$E(R_A)$	=	expectation returns of stock A.
$E(R_B)$	=	expectation returns of stock B.
N	=	total historical data observations for large samples (minimum 30 observations) and for small samples use (n-1).

The Equations for calculations, the method to be used, are formed based on the Markowitz model approach. The goal to be achieved is to minimize risk while the limit used is the magnitude of return of each share. The calculated risk is the difference value of each share. Return is the expected return value of each share which is expected to be greater than the average interest rate of Bank Indonesia Certificates.In the quadratic programming method, after the equation and its constraints are formed, it makes a simplex table then uses iteration to find the optimal proportion.

The stocks studied were LQ45 index. The research method used in making this research is descriptive and verification methods. To conduct data analysis, the steps and methods are as follows:

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a. The sample is taken from the population, namely all shares listed on the Indonesia Stock Exchange. The sample in this study is stocks that are included in the LQ45 group from February 2004 to July 2009, consecutively. The companies registered during these periods are as follows:

Table 1. List of Stocks That Enter consecutively in the Calculation of the LQ45 Index. Period February 2004 - July 2009

No	Code	Emiten Name
1	AALI	Astra Agro Lestari Tbk
2	ANTM	Aneka Tambang (Persero) Tbk
3	ASII	Astra International Tbk
4	BBCA	Bank Central Asia Tbk
5	INCO	International Nickel Indonesia Tbk
6	INDF	Indofood Sukses Makmur Tbk
7	INKP	Indah Kiat Pulp & Paper Tbk
8	ISAT	Indosat Tbk
9	PTBA	Tambang Batubara Bukit Asam Tbk
10	SMCB	Holcim Indonesia Tbk
11	TLKM	Telekomunikasi Indonesia Tbk
12	UNTR	United Tractors Tbk

Source : PT. Bursa Efek Jakarta

b. Building a nonlinear programming model and developing an optimal portfolio. The method used in doing this optimization is the quadratic programming method.

In the QP calculation, the diversity of limits owned by investors can be used in determining portfolio optimization. There is a lot of research on portfolio optimization but very few use QP. In this study, how to do portfolio optimization using the limitation besides the Markowitz model is also the hope that the return on investment can be more than the interest rate. The interest rate used is the Indonesian interest rate (SBI).

RESULTS AND DISCUSSION

Based on the Markowitz model, the objective function to be achieved is to minimize risk. The values in the objective function are the risk values obtained from the value of the variance of each stock. The limits to be used based on the Markowitz model approach are as follows:

1. The total of proportion invested in each asset for all n assets is equal to 1
2. The average amount of each return is greater than the interest rate of Bank Indonesia Certificates. Data return used is data return from monthly geometric averages. Based on the calculation results, the SBI geometric average is 8.88% per year or 0.74% per month.
3. The proportion of each security must not be negative so it will have a value greater than 0

In previous studies, the formation of portfolio optimization can be used by means of liner programming (LP). The difference between LP and QP is the objective function where QP has a quadratic objective function because the results formed are non-linear, not linear. Based on the stock price of the companies in table 1, we calculate the covariance so that the following results are obtained:

Objective Function

Minimize

Z =	0.0226	$X1^2$	+	0.0036	X1.X2	+	0.0128	X1.X3	+	0.0098	X1.X4	+
	0.0011	X1X5	+	0.0017	X1.X6	+	0.0084	X1.X7	+	0.0123	X1X8	+
	0.0102	X1.X9	+	0.0111	X1X10	+	0.0007	X1.X11	+	0.0107	X1X12	+
	0.0092	X2.X2	+	0.0008	X2.X3	+	0.0030	X2X4	+	0.0021	X2X5	+
	-0.0008	X2.X6	+	0.0042	X2.X7	+	0.0039	X2.X8	+	0.0035	X2.X9	+
	0.0033	X2.X10	+	-0.0004	X2.X11	+	0.0070	X2.X12	+	0.0327	$X3^2$	+
	0.0106	X3.X4	+	0.0009	X3.X5	+	0.0020	X3.X6	+	0.0084	X3.X7	+
	0.0109	X3.X8	+	0.0057	X3.X9	+	0.0097	X3.X10	+	0.0031	X3.X11	+
	0.0094	X3.X12	+	0.0165	$X4^2$	+	0.0023	X4.X5	+	-0.0002	X4.X6	+
	0.0094	X4.X7	+	0.0116	X4.X8	+	0.0070	X4.X9	+	0.0104	X4.X10	+
	-0.0002	X4.X11	+	0.0080	X4.X12	+	0.1300	$X5^2$	+	0.0066	X5.X6	+
	0.0015	X5.X7	+	0.0094	X5.X8	+	0.0069	X5.X9	+	0.0022	X5.X10	+
	-0.0012	X5.X11	+	-0.0010	X5.X12	+	0.0038	$X6^2$	+	0.0001	X6.X7	+
	-0.0008	X6.X8	+	-0.0007	X6.X9	+	0.0012	X6.X10	+	0.0001	X6.X11	+
	-0.0010	X6.X12	+	0.0124	$X7^2$	+	0.0094	X7.X8	+	0.0074	X7.X9	+
	0.0101	X7.X10	+	-0.0001	X7.X11	+	0.0093	X7.X12	+	0.0347	$X8^2$	+
	0.0112	X8.X9	+	0.0111	X8.X10	+	0.0004	X8.X11	+	0.0093	X8.X12	+
	0.0150	$X9^2$	+	0.0092	X9.X10	+	-0.0003	X9.X11	+	0.0077	X9.X12	+
	0.0158	$X10^2$	+	0.0003	X10.X11	+	0.0069	X10.X12	+	0.0043	$X11^2$	+
	0.0000	X11.X12	+	0.0188	$X12^2$							

Based on the Markowitz model approach to the limits described above, then the shape of limitations are as follows:

with limits

X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 = 1

0.0456	X1	+	0.0055	X2	+	0.0135	X3	+	0.0152	X4	+	
0.0570	X5	+	0.0242	X6	+	0.0280	X7	+	0.0035	X8	+	
0.0400	X9	+	0.0340	X10	+	0.0125	X11	+	0.0182	X12	≥	0.0074

X1, X2, X3, X4, X5, X6, X7, X8, X9, X10, X11, X12 ≥ 0

where

X1 :	Tambang Batubara Bukit Asam Tbk Shares	(PTBA)
X2 :	Indosat Tbk Shares	(ISAT)
X3 :	Indah Kiat Pulp & Paper Tbk Shares	(INKP)
X4 :	Indofood Sukses Makmur Tbk Shares	(INDF)
X5 :	International Nickel Indonesia Tbk Shares	(INCO)
X6 :	Bank Central Asia Tbk Shares	(BBCA)
X7 :	Astra International Tbk v Shares	(ASII)

X8 :	Aneka Tambang (Persero) Tbk Shares	(ANTM)
X9 :	Astra Agro Lestari Tbk Shares	(AALI)
X10 :	United Tractors Tbk Shares	(UNTR)
X11 :	Telekomunikasi Indonesia Tbk Shares	(TLKM)
X12 :	Holcim Indonesia Tbk Shares	(SMCB)

In solving the quadratic problem, LINGO is used in the calculation process. Based on the calculation results, the investment that should be done according to the optimization of the quadratic programming method is onIndosat Tbk (ISAT) of 13,2%, Indofood Sukses Makmur Tbk (INDF) of 4,12%, Bank Central Asia Tbk(BBCA) of 37,55%, Astra International Tbk (ASII) of 4,11%, Aneka Tambang (Persero) Tbk (ANTM) of 0,91%, Astra Agro Lestari Tbk (AALI) of 5,73%, Telekomunikasi Indonesia Tbk (TLKM) of 31,9% dan Holcim Indonesia Tbk (SMCB) of 2,45%. Risks arising from investments and the limits are 0.14%.

CONCLUSIONS AND SUGGESTIONS

Conclusions

The optimal investment composition is based on the calculation of the quadratic programming method using LINGO, that is by investing in stocks Indosat Tbk (ISAT) of 13,2%, Indofood Sukses Makmur Tbk (INDF) of 4,12%, Bank Central Asia Tbk(BBCA) of 37,55%, Astra International Tbk (ASII) of 4,11%, Aneka Tambang (Persero) Tbk (ANTM) of 0,91%, Astra Agro Lestari Tbk (AALI) of 5,73%, Telekomunikasi Indonesia Tbk (TLKM) of 31,9% dan Holcim Indonesia Tbk (SMCB) of 2,45%. Based on the composition of the investment, then the investment will have a risk of 0.14% within the existing limits.

Suggestions

The use of LINGO in doing calculations is very good, but in the process of writing the formulation needs to be careful, especially when it comes to using a very large number of research variables.

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3. LQ45 Index

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