ON THE EXTREME POSITION OF PROPAGATION OF BICHROMATIC WAVE IN HYDRODYNAMIC LABORATORY

PERAMBATAN GELOMBANG *BICHROMATIC* PADA POSISI EXTRIM DI LABORATORIUM *HYDRODYNAMIC*

Marwan
Department of Mathematics, FMIPA, Syiah Kuala University
Jl. Tgk. Syech Abdurrauf Darussalam-Banda Aceh, Indonesia, 23111
email : marwan.ramli@math-usk.org

ABSTRACT

This paper concerns with the down-stream propagation of waves over initially still water. Such study is relevant to generate waves of large amplitude in wave tanks of a hydrodynamic laboratory. Input in the form of a time signal is provided at the wave-maker located at one side of the wave tank; the resulting wave then propagates over initially still water towards the beach at the other side of the tank. Experiments show that nonlinear effects will deform the wave and may lead to large waves with wave heights larger than twice the original input; the deformations may show it as peaking and splitting. It is of direct scientific interest to understand and quantify the nonlinear distortion; it is also of much practical interest to know at which location in the wave tank, the extreme position, the waves will achieve their maximum amplitude and to know the amplitude amplification factor. To investigate this, a previously introduced concept called Maximal Temporal Amplitude (MTA) is used: at each location the maximum over time of the wave elevation. An explicit expression of the MTA cannot be found in general from the governing equations and generating signal. The model is used in this paper is a Korteweg-de Vries (KdV) model and third order approximation theory to calculate the approximate extreme positions for a class of wave. This class is the wave-groups that originate from initially bi-chromatics type of wave, described by superposition of two monochromatic waves. It is shown that, the extreme position depends on the amplitude and the wave length of the group. The theoretical results are verified with numerical as well as experimental results for comparison.
Keywords : Nonlinear distortion, Maximal Temporal Amplitude, extreme position, bi- chromatics waves, KdV equation, third order approximation.

ABSTRAKSI

Tulisan ini terkonsentrasi pada perambatan gelombang air di laboratorium. Kajian ini dilakukan karena kebutuhan laboratorium untuk membangkitkan gelombang beramplitudo tinggi di kolam pengujian. Hasil eksperimen menunjukkan bahwa sebagai akibat ketaklinieran medium air menyebabkan bahwa gelombang air mengalami perubahan bentuk yang ditandai dengan peristiwa pemuncakan dan pembelahan. Terkait dengan hal ini adalah penting untuk diketahui di posisi mana gelombang tersebut mengalami pemuncakan tertinggi (posisi ekstrim) dan berapa pula besarnya kenaikan amplitudo gelombang dibandingkan dengan amplitudo gelombang di posisi awal. Untuk menyelidiki hal tersebut diperkenalkan suatu kuantitas yang disebut dengan Maximal Temporal Amplitude (MTA). Kuantitas ini digunakan untuk mengukur ketinggian gelombang pada suatu posisi untuk setiap waktu. Ekpresi eksplisit kuantitas ini tidak dapat ditemukan apabila pembangkitan gelombang dilakukan dengan menggunakan persamaan lengkap. Dalam tulisan ini digunakan persamaan Korteweg-de Vries (KdV) dan pendekatan orde ketiga untuk menghitung posisi ekstrim gelombang yang pada awalnya berupa gelombang bikromatik. Gelombang ini merupakan superposisi dua gelombang monokromatik beramplitudo sama tetapi dengan frekuensi yang berbeda. Terlihat bahwa posisi ekstrim bergantung pada amplitudo dan panjang gelombang gelombang bikromatik dimaksud. Hasil ini serupa dengan hasil eksperimen dan hasil numerik .
Kata kunci : distorsi, Maximal Temporal Amplitude, posisi ekstrim, gelombang bikromatik,persamaan KdV, aproksimasi orde ketiga

INTRODUCTION

This study is directly motivated to be able to generate extreme waves in wave tanks of hydrodynamic laboratories. In such a generation, a time signal is given to a wave-maker that determines the motion of flaps that push the water. Waves are then produced that propagate downstream over initially still water along the wave tank. Because of non-linear effects the original signal deforms, see (Andonowati, et all, 2004), (Groesen, et all, 1999), (Stansberg, 1998), (Westhuis, et all, 2000) and (Westhuis, et all, 2001). This nonlinear deformation may lead to amplification of the waves so that waves can occur with wave heights that cannot be generated in a direct way by wave-maker motions.

The nonlinear effects are difficult to study over the long distances and times that are relevant for the laboratory. In particular it is not clear which waves, i.e. which waves resulting from a certain signal, will show amplitude amplification; and if so, at which position in the tank the largest waves will appear. This position is called the extreme position. The inverse problem, given a specified extreme position, find the possible generating wave maker signals, is of most scientific interest, and of direct relevance for the hydrodynamic laboratories.

To investigate these problems, it is most fruitful to interpret the downstream evolution as a spatial succession of wave signals. Directly related to this is a concept called the Maximum Temporal Amplitude (MTA) that has been introduced in (Andonowati and Groesen, 2003). The MTA measures at each location in the wave tank the maximum over time of the surface elevation. The location where the MTA curve achieves its maximum is the extreme position, there the largest waves will be found. The time signal of the surface elevation at that position will be called the extreme signal. Clearly this signal depends on the input at the wave maker. The ratio of the maximal value of the MTA compared to its value at the wave-maker defines the amplification factor. The MTA can also be used for the inverse problem when a wave-field with a clear extreme position is considered. Suppose that *LD* is the distance in the wave tank from the wave-maker where the extreme signal is requested to appear. If the extreme position of the wave-field is denoted by x_{max}, the signal to be generated at the wave-maker should be the signal of the wave-field at the location of $x = x_{max}$.

The aim of this paper is to derive the approximate extreme positions for certain wave-fields at the wave-maker. The wave fields that arise from bi-chromatics wave maker signal are restricted. Such signal consist of two mono-chromatic components of the same amplitudes but slightly different of frequencies. This signal develops into highly distorted wave-groups with clear amplitude amplification from nonlinear effects. They are prototypes of interesting wave groups with extreme waves; see for instance the insightful investigations in Longuet (1984), Phillips (1993) and Donelan (1990). For this group, many numerical and experimental results are available to verify our theoretical investigation, such as (Westhuis, et all, 2000) and (Westhuis, et all, 2001).

To simplify the technical calculations in the following, instead of the full surface wave equations, it will use a modified Korteweg-de Vries (KdV) model with exact dispersion relation; for the kind of waves under consideration, this is a valid approximation. As has been shown in previous work, see also (Andonowati and Groesen, 2003), for narrow banded spectra, the third order effects can dominate the second order effects and are responsible for the large amplification factors. For that reason in analysis is used third order theory.

The organization of the paper is as follows. The next session present the mathematical model to be used and the third order asymptotic expansion for this model. The approximation method to the MTA and the extreme position by an explicit expression obtained from third order theory are discussed in Section 3. The results and the verification of the derived formulas for the extreme position are presented in Section 4. Finally, Section 5 presents some concluding remarks.

RESEARCH PROCEDURE

Third order theory for the KdV model

The evolution of rather long and rather small surface gravity waves is governed to a reasonable approximation by the well known KdV equation. In normalized variables, the KdV equation with full dispersion (Groesen, 1998) has the form

$$\partial_t \eta + i\Omega(-i\partial_x)\eta + \frac{3}{2}\eta\partial_x\eta = 0 \qquad (1)$$

with $\eta(x,t)$ is the surface elevation and Ω is the operator that produces the dispersion relation between frequency ω and wave number k for small amplitude waves given by

$$\omega = \Omega(k) = \sqrt{k \tanh k}$$

The laboratory variables for the wave elevation, horizontal space and time $\eta_{lab}, x_{lab}, t_{lab}$ are related to the normalized variables by $\eta_{lab} = \eta h, x_{lab} = xh, t_{lab} = t\sqrt{h/g}$, with h is the uniform water depth and g is the gravity acceleration. Consequently, corresponding trans-formed wave parameters such as wave length, wave number and angular frequency, are given by $\lambda_{lab} = \lambda h, k_{lab} = \frac{k}{h}, \omega_{lab} = \omega\sqrt{g/h}$.

In this paper, the solution of (1) is approximated by using a direct expansion up to third order in the power series of the wave elevation. Here, it is written as

$$\eta \approx \varepsilon\eta^{(1)} + \varepsilon^2\eta^{(2)} + \varepsilon^3\eta_{sb}^{(3)} \qquad (2)$$

with ε is a positive small number representing the order of magnitude of the wave amplitude. The terms $\eta^{(1)}, \eta^{(2)}$ and $\eta^{(3)}$ describe the linear first order, the second and third order nonlinear term, respectively. Assuming that the linear term $\eta^{(1)}$ consists of three frequencies that are close to each other (narrow band), in the third order we take only the largest contribution, namely the third order side band $\eta_{sb}^{(3)}$. The frequencies of the side bands are close to the frequency of the linear term. It is known that this direct expansion leads to resonance in the third order, see [(Andonowati and Groesen, 2003), (Cahyono, 2002) and (Marwan and Andonowati, 2003)]. To prevent the resonant term, this expansion modify using Linstead-Poincare technique (Whitham, 1974) by allowing a nonlinear modification of the dispersion relation

$$k = k^{(0)} + \varepsilon k^{(1)} + \varepsilon^2 k^{(2)} \qquad (3)$$

with $k^{(0)} = \Omega^{-1}(\omega)$.

For the linear signal we take

$$\eta^{(1)} = \sum_{p=1}^{N} a_p e^{i\phi_p} + c.c, \qquad (4)$$

with $N = 2$. Here, $\phi_p = k_p x - \omega_p t + \psi_p$, where (k_p, ω_p) are related by the linear dispersion relation, and with ψ_p the phase of each mono-chromatic wave, $c.c$ denotes the complex conjugate of the previous terms. The following procedure has been described in (Andonowati, et all, 2003), (Cahyono, 2002) and (Marwan and Andonowati, 2003) without taking the phases of the mono-chromatic components of the signals into account, i.e. for $\psi_p = 0$. Since this paper aim to investigate the effect of these mono-chromatic phases on the global behavior of the propagating signal along the wave tank, it add arbitrary phase ψ_p.

Substituting (2) and (3) into (1), for $\eta^{(1)}$ as the linearized solution as in (4), the second order leads to $k_p^{(1)} = 0$ $p = 1, \dots, N$ and

$$\eta^{(2)} = \frac{3}{2}\sum_{p=1}^{N}\sum_{q=1}^{N} a_p a_q \begin{bmatrix} s_+ e^{i(\phi_p+\phi_q)} \\ +s_- e^{i(\phi_p-\phi_q)} \end{bmatrix} + c.c, \qquad (5)$$

with

$$s_\pm = \frac{1}{2}\frac{k_p^{(0)} \pm k_q^{(0)}}{\omega_p \pm \omega_q - \Omega(k_p^{(0)} \pm k_q^{(0)})}.$$

In order to distinguish the free waves that will be introduced later, the second order solution in (5) call as the second order bound wave; this solution contains non-linear terms as the results of mode generation. The resonant terms in the third order bound wave, lead to the non-linear dispersion relation

$$k_p^{nl} = k_p^{(0)} + k_p^{(2)}$$

with

$$k_p^{(2)} = -\frac{9}{4}\frac{k_p^{(0)}}{V_g(k_p^{(0)})}\left[\sum_{q=1}^{N} a_q^2 (s_+ + s_-)\right]. \qquad (6)$$

The third order side band $\eta_{sb}^{(3)}$ can be expressed as

$$\eta_{sb}^{(3)} = \frac{9}{4}\sum_{p=q}^{N}\sum_{q\neq r}^{N}\sum_{r=1}^{N} \frac{a_p a_q a_r L_{pq,r}}{(s_p + s_{q,r})e^{i(\phi_p+\phi_q-\phi_r)}} + \frac{9}{8}\sum_{p=q}^{N}\sum_{p\neq q,q\neq r}^{N}\sum_{r=1}^{N} \frac{a_p a_q a_r L_{pq,r}}{(s_{p,r} + s_{q,r})e^{i(\phi_p+\phi_q-\phi_r)}} + c.c, \qquad (7)$$

with

$$s_p = \frac{k_p^{(0)}}{2\omega_p - \Omega\left(k_p^{(0)}\right)},$$

$$s_{pq} = \frac{1}{2}\frac{k_p^{(0)} + k_q^{(0)}}{\omega_p + \omega_q - \Omega\left(k_p^{(0)} + k_q^{(0)}\right)},$$

$$s_{p,r} = \frac{k_p^{(0)} - k_q^{(0)}}{\omega_p - \omega_q - \Omega\left(k_p^{(0)} - k_q^{(0)}\right)},$$

$$L_{pq,r} = \frac{k_p^{(0)} + k_q^{(0)} - k_r^{(0)}}{\omega_p + \omega_q - \omega_r - \Omega\left(k_p^{(0)} + k_q^{(0)} - k_r^{(0)}\right)},$$

And

$$V_g\left(k_p^{(0)}\right) = \Omega'\left(k_p^{(0)}\right), \; p = 1,2,\dots N.$$

This paper mimic the generation of waves in a hydrodynamic laboratory and there are interested in a solution that at a given position, say $x = 0$, is given by the signal

$$\eta^{(1)}(0,t) = \sum_{p=1}^{N} a_p e^{i\theta_p} + c.c \qquad (8)$$

with $\theta_p = \omega_p t - \psi_p$. To satisfy the signal at this position, the contribution of the second order and third order side band terms at $x = 0$ have to be compensated by harmonic modes, called free waves. The second order free waves are given by

$$\eta_{free}^{(2)} = \frac{3}{2}\sum_{p=1}^{N}\sum_{q=1}^{N} a_p a_q \begin{bmatrix} s_+ e^{i\vartheta(\omega_p+\omega_q)} \\ +s_- e^{i\vartheta(\omega_p-\omega_q)} \end{bmatrix} + c.c. \qquad (9)$$

This is a wave with the same frequencies as in the second order bound wave, but consisting of harmonic modes that satisfy the linear dispersion relation. The third order side band free wave consists similarly of monochromatic waves and is of the form

$$\eta_{sb}^{(3)} = \frac{9}{4}\sum_{p=q}^{N}\sum_{q\neq r}^{N}\sum_{r=1}^{N} \frac{a_p a_q a_r L_{pq,r}}{(s_p + s_{q,r})e^{i\vartheta(\omega_p+\omega_q-\omega_r)}} + \frac{9}{8}\sum_{p=q}^{N}\sum_{p\neq q,q\neq r}^{N}\sum_{r=1}^{N} \frac{a_p a_q a_r L_{pq,r}}{(s_{p,r} + s_{q,r})e^{i\vartheta(\omega_p+\omega_q-\omega_r)}} + c.c, \qquad (10)$$

with $\vartheta(\omega_p) = \Omega^{-1}(\omega_p)x - \omega_p t + \psi_p$. Taken together, the third order solution of (1) and satisfying (8) is

$$\tilde{\eta} \approx \varepsilon\eta^{(1)} + \varepsilon^2\eta^{(2)} - \varepsilon^2\eta_{free}^{(2)} + \varepsilon^3\eta_{sb}^{(3)} - \varepsilon^3\eta_{sb,free}^{(3)}. \qquad (11)$$

Maximal Temporal Amplitude and the extreme position

In previous studies of bichromatics waves (Groesen, et all, 1999), (Stansberg, 1998), (Westhuis, et all, 2000) and (Westhuis, et all, 2001), it was found experimentally, numerically and theoretically that depending on the wave amplitude, but just as well as on the frequency difference, large deformations and amplitude increase can develop. This was made more clearly visible in (Andonowati and Groesen, 2003), where, for the corresponding optical problem, the so-called *maximal temporal amplitude* (MTA) was introduced. At each position downstream from the wave generator, this MTA measures the maximum over time of the surface elevation. When plotted as a curve in the downstream direction, this curve shows (almost periodic) oscillatory behaviour in which several wavelengths can be seen and interpreted. At specific locations the curve achieves its maximum at places where the amplitude amplification (compared to the amplitude of the generated wave) is maximal and where `extreme' waves appear. According to the previous section, the third order approximated solution of (1) is given by (11), we will use this approximation to study the MTA, and hence we take the MTA to be defined as

$$m(x) = \max_t \tilde{\eta}(x,t). \qquad (12)$$

In deterministic extreme wave generation performed in hydrodynamic laboratories, MTA is proved to be a useful concept to predict the position where the most extreme signal appears in the wave tank, (Andonowati, et all, 2004). Further-more, it gives a practical value of the Amplitude Amplification Factor, AAF = $m(x_{max}) = m(0)$, with x_{max} is the first position where $m(x_{max}) = \max_x m(x)$, for $0 < x < L$ and L is the length of the wave tank. Therefore it is of interest to calculate the value of x_{max} and the dependence of this position on the input signal at the
wave maker.

In (Andonowati and Groesen, 2003) an explicit expression was given for x_{max} for input signals in the form of bichromatics waves with zero phases for a KdV model of an optical pulse propagation in non-linear media using third order approximation. A similar formulation for bi-chromatics is derived in (Marwan and Andonowati, 2003) for propagation of water wave where the initial phases of the mono-chromatic components is zero.

In the following the derivation the explicit expression of x_{max} for an input signal in the form of bi-chromatics wave are briefly. It is shown that this x_{max} does not depend on the initial phases of the mono-chromatic components. Denoting the indices 1 and 2 in the expressions (4) - (10) by (+) and (-) for the bi-chromatics signals, the linear term can be written as

$$\eta^{(1)} = q[\cos\theta_+ + \cos\theta_-], \qquad (13)$$

with $\theta_{\mp} = k_{\mp}^{(0)} x - \omega_{\mp} t + \psi_{\mp}$, $\psi_{\mp}$ the phase of each mono-chromatic wave, and the wave numbers ordered according to $k_{+}^{(0)} > k_{-}^{(0)}$.

The following expressions are rewritten from (5) - (10).

$$\eta^{(2)} = \frac{9}{4} q^2 \begin{bmatrix} s_+ \cos 2\theta_+ + s_- \cos 2\theta_- \\ +2s \cos(\theta_+ + \theta_-) \\ +s_0 \cos(\theta_+ - \theta_-) \end{bmatrix} \quad (14)$$

$$\eta_{free}^{(2)} = \frac{9}{4} q^2 \begin{bmatrix} s_+ \cos\vartheta(2\omega_+) \\ +s_- \cos\vartheta(2\omega_-) \\ +2s \cos\vartheta(\omega_+ + \omega_-) \\ +s_0 \cos\vartheta(\omega_+ - \omega_-) \end{bmatrix} \quad (15)$$

$$k_{\mp}^{(2)} = -\frac{9}{4} q^2 \frac{k_{\mp}^{(0)}}{V_g(k_{\mp}^{(0)})} [s_0 + s_{\mp} + 2s] \quad (16)$$

Using this wave number correction, the first order expansion $\eta^{(1)}$ can be written as

$$\eta^{(1)} = 2q \cos(\bar{k}^{nl} x - \bar{\omega} t + \bar{\psi}) \cos(\kappa^{nl} x - \nu t + \Delta\psi) \quad (17)$$

with

$\bar{\omega} = \frac{\omega_+ + \omega_-}{2}, \nu = \frac{\omega_+ - \omega_-}{2}, \bar{\psi} = \frac{\psi_+ + \psi_-}{2}, \Delta\psi = \frac{\psi_+ - \psi_-}{2}, \kappa = \Omega^{-1}(\nu)$

.

The third order bound and free waves are expressed as

$$\eta_{sb}^{(3)} = q^3 B_+ \cos(\bar{k}^{nl} x - \bar{\omega} t + \bar{\psi}) \cos(3\kappa^{nl} x - 3\nu t + 3\Delta\psi) - q^3 B_- \sin(\bar{k}^{nl} x - \bar{\omega} t + \bar{\psi}) \sin(3\kappa^{nl} x - 3\nu t + 3\Delta\psi) \quad (18)$$

and

$$\eta_{sb,fw}^{(3)} = q^3 B_+ \cos(\bar{K} x - \bar{\omega} t + \bar{\psi}) \cos(\tilde{K} x - 3\nu t + 3\Delta\psi) - q^3 B_- \sin(\bar{K} x - \bar{\omega} t + \bar{\psi}) \sin(\tilde{K} x - 3\nu t + 3\Delta\psi) \quad (19)$$

with $\bar{k}^{nl}$ and κ^{nl} are nonlinear wave numbers, corresponding to ω and ν respectively,

$$\bar{K} = (\Omega^{-1}(2\omega_+ - \omega_-) + \Omega^{-1}(2\omega_- - \omega_+))/2,$$

$$\tilde{K} = (\Omega^{-1}(2\omega_+ - \omega_-) - \Omega^{-1}(2\omega_- - \omega_+))/2$$

The coefficients of third order side bands are $B_{\pm} = a_+ \pm a_-$, with

$$a_+ = \frac{9}{4}(s_+ + s_0) \frac{2k_+^{(0)} - k_-^{(0)}}{2\omega_+ - \omega_- - \Omega(2k_+^{(0)} - k_-^{(0)})}$$

and

$$a_- = \frac{9}{4}(s_- + s_0) \frac{2k_-^{(0)} - k_+^{(0)}}{2\omega_- - \omega_+ - \Omega(2k_-^{(0)} - k_+^{(0)})},$$

see (Marwan and Andonowati, 2003).

For wave parameters of laboratory interest $B_- << B_+$, the expressions (17) and (18) show that the first and the third order side band bound wave have approximately the same carrier. The superposition of the first order with the third order side band bound and free waves leads to a spatial envelope of the carrier wave, resulting in a modulation of the carrier. Under the assumption $B_- << B_+$, the phases of first order, third order side band bound and free waves are the same, namely $\bar{\psi}$, and so the spatial envelope has modulation length $\lambda = 4\pi/|\bar{k}^{nl} - \kappa^{nl}|$. This value can in fact be obtained by considering the superposition of the third order bound waves and free waves only where λ appears as the 'wave length' of MTA. The positions of zero phase of the spatial envelope show that the location of the first maximum of MTA does not depend on the phases of mono-chromatic components and it can be expressed in the form

$$\check{x}_{max} \approx \frac{\pi}{|\bar{k}^{nl} - \kappa^{nl}|} = \frac{\pi}{\left|2\nu^2 \hat{\beta} + 2q^2 \left(\hat{\gamma} + \frac{\frac{9}{4}\sigma_2 \bar{k}}{V_g(\bar{k})}\right)\right|} = O\left(\frac{1}{q^2}\frac{1}{\nu^2}\right), \quad (20)$$

with

$$\hat{\beta} = -\frac{\Omega''(\bar{k})}{2V_g^3(\bar{k})},$$

$$\hat{\gamma} = \frac{9}{4} \frac{\bar{k}}{V_g(\bar{k})} \left[\frac{1}{V_g(\bar{k}) - 1} + \frac{\bar{k}}{2\bar{\omega} - \Omega(2\bar{k})}\right],$$

$$\sigma_2 = \frac{\bar{k}}{2\bar{\omega} - \Omega(2\bar{k})}.$$

RESULTS AND DISCUSSION

Verification of the derived formulas

In what follows, the formulas derived in Section 3 are verified. Here, it is used the available numerical and experimental results. For the experiments, the propagated signals are measured only at a limited number of locations in the wave tank, hence, the location where the MTA is maximal, the extreme position, can only be obtained within a range determined by the locations where the signals are measured.

The variables are used here in standard SI units [m, s]. A typical wave tank with a layer of water of 5m deep, and with a length of 250m, and express all quantities in laboratory variables are considered. The predicted extreme position x_{max} derived by the third order approximation (TOA) in (20) will be compared with results from a numerical wave tank, HUBRIS, used at Maritime Research Institute Netherlands (MARIN) (Westhuis, et. al., 2001) and with experimental results reported in (Stansberg, 1998) and (Westhuis, et al., 2001). We present the results in Table 1, providing the reference to the experiments. It is seen that the predicted values are reasonably close to both the numerical values as well as to the experimental results.

Table 1. Comparisons of the extreme positions $\breve{x}_{max}$ calculated with third order theory (20) above, the numerical results $x_{max}(\text{HBR})$ calculated numerically, and the experimental result $x_{max}(\text{EXP})$ with specification of the various cases listed in the given references.

Case	x_{max} (HBR)	$\breve{x}_{max}$	x_{max} (EXP)
$q = 0.09$, $\omega_+ = 3.264$, $\omega_- = 3.028$	155.0 m	157.5 m	$140 - 160$ m (Westhuis, et all, 2001)
$q = 0.08$, $\omega_+ = 3.30$, $\omega_- = 2.99$	127.0 m	118.0 m	$100 - 120$ m (Stansberg, 1998) and (Westhuis, et all, 2001)
$q = 0.09$, $\omega_+ = 3.30$, $\omega_- = 2.99$	109.8 m	110.9 m	$100 - 120$ m (Westhuis, et all, 2001)
$q = 0.1$, $\omega_+ = 3.491$, $\omega_- = 2.856$	47.0 m	48.62 m	$40 - 60$ m (Westhuis, et all, 2001)

Figure 1 present MTA curve computed numerically with HUBRIS for an input bichromatics signals with amplitude, $q = 0.08$, and frequencies of the monochromatics $\omega_+ = 3.30$ and $\omega_- = 2.99$. It can be seen that this signal deform in its propagation. The signal has amplitude 0.16 m at the certain position (*x=0*) and 0.32 m at the extreme position (*x=127* m). So, the amplification factor of this signal is 2. Experiments for this case have been conducted independently in (Stansberg, 1998) and (Westhuis, et all, 2001) where in both experiments the largest signal appears at a distance of approximately 120m away from the wave-maker, which is the extreme position. Figure 2 show signals at different locations in the wave tank computed using HUBRIS. As shown in Table 1, for this case the third order approximation in (20) gives the position of $\breve{x}_{max} = 118$ m away from the wave maker.

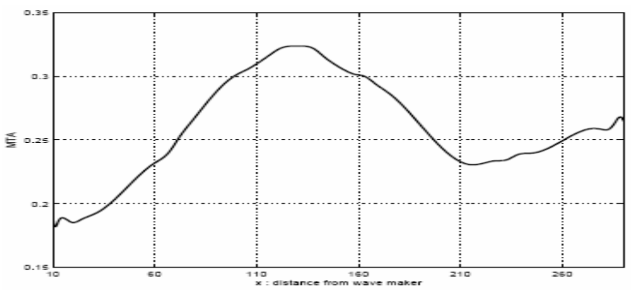

Figure 1. MTA for bi-chromatics signal with $q = 0.08, \omega_+ = 3.30$, and $\omega_- = 2.99$

CONCLUDING REMARKS

This paper has considered propagation in initially still water of waves generated at a wave maker by a bi-chromatics signal. This class contain signals having two mono-chromatic components of the same amplitudes but slightly different of frequencies. While propagating downstream from the wave maker, the input signals changes in shape and amplitude, in the considered cases, a largely amplified elevation is found somewhere down stream in the tank. By using MTA, location of the maximum of surface elevation over time can be defined. Furthermore, prescribing the position where the extreme wave has to appear in the wave tank, the MTA can also be used to assist what kind of signal has to be generated at the wave-maker in such a way that the propagating signal produces the requested extreme wave elevation at the requested position. The comparisons of this result to both experimental and numerical results show reasonably close values of the predicted locations and the known results. Future works will focus on Benjamin-Feir wave group propagation and apply similar methods used in this paper.

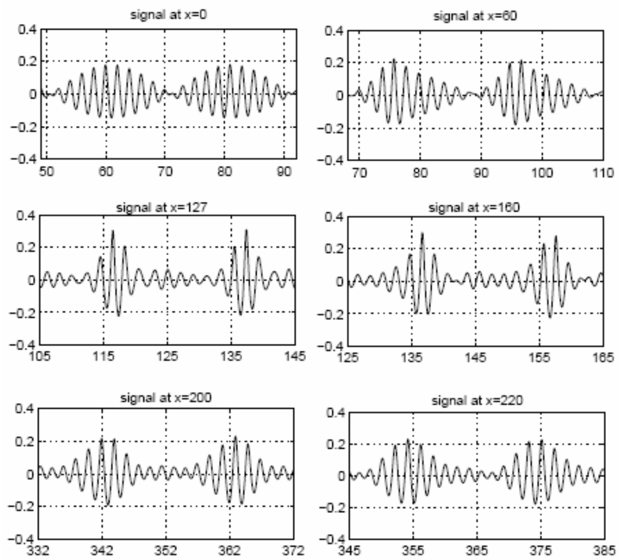

Figure 2. Bi-chromatics signal at some positions with $q = 0.08, \omega_+ = 3.30$, and $\omega_- = 2.99$ computed using HUBRIS.

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