APPLICATION OF FORWARD – BACKWARD PROPAGATION ALGORITHM FOR THREE – PHASE POWER FLOW ANALYSIS IN RADIAL DISTRIBUTION SYSTEM

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Abstrak

Power flow analysis is backbone of power system analysis and design. It is essential for power system analysis in operational as well as in planning stages. Power flow calculation is normally performed by simply considering that the system under study is completely balance. Actual distribution system is inherently unbalance and therefore three-phase power flow is necessary to accurately simulate the unbalance system. However, inclusion of unbalance increases the dimension of problem and, as distribution system is commonly constructed as radial system with high R/X ratio, it may also cause the sophisticated power flow algorithms fail to converge. The robust algorithm for three-phase power flow is therefore needed. In this paper, three-phase power flow for unbalance distribution system is carried out using Forward-Backward Propagation Technique. The equivalent injection current method is employed to represent the loads and shunt admittances. The algorithm starts with mapping the distribution network to determine the forward and backward propagation paths. The backward propagation is used to calculate branch currents using the bus injection currents. The forward propagation is employed to calculate bus voltages using the obtained branch currents and line impedances. The algorithm offers robust and good convergence characteristics for radial distribution system. The algorithm is presented for the IEEE 34-bus system with satisfied generated results.

Keyword*: Convergence; equivalent injection current; forward-backward propagation; unbalance system*

Introduction

Power flow calculation is backbone of power system analysis and design. It is essential for analysis of any power system in the operational as well as planning stages. The calculation is initially carried out by formulating the network equation. Node-voltage method, which is the most suitable form for many power system analyses, is commonly used. Mathematically, power flow problem requires solution of simultaneous nonlinear equations and normally employs an iterative method, such as Gauss-Seidel and Newton-Raphson.

Power flow calculation is normally performed by simply considering that the system under study is balance. Hence, the calculations are carried out for single phase assuming that the other two phases are exactly the same but with the 120 degrees phase difference. The asymmetry in lines and loads produces a certain level of unbalance in real power system and this is considered as disturbance whose level should be controlled to maintain the electromagnetic compatibility of the system (Mayordomo, Izzeddine et al. 2002).

For distribution system, in particular, the system is inherently unbalanced, due to factors such as the unbalance of customer loads, the presence of unsymmetrical line spacing, and the combination of single, double and three-phase line sections. Therefore, three-phase power flow is necessary to accurately simulate the unbalance system.

Inclusion of unbalance increases the dimension of problem as all the three phases need to be considered instead of single phase balanced representation. On the other hand, distribution system commonly constructed as radial system with high R/X ratio may cause the sophisticated power flow algorithms fail to converge. The robust algorithm for three-phase power flow is therefore needed.

Decomposition of the coupled unbalance system into positive, negative and zero symmetrical components is the most popular approach used in three-phase power flow algorithms (Lo and Zhang 1993; Zhang and Chen 1994). This eliminates the mutual coupling between phases so a three-phase power flow can be run three times, once for each phase. However, if the coupling between sequences occurs, then there is no real advantage in decomposing the system into the symmetrical components. Furthermore, this may result in significant error in calculation.

Another approach is decoupling the three-phase system into individual phases by introducing compensation current injections (Chen, Chen et al. 1991; Cheng and Shirmohammadi 1995; Lin and Teng 2000; Vieira, Freitas et al. 2004). Therefore, a three-phase power flow can be solved independently for every phase without utilization of symmetrical components. All components are modeled by phase voltages, admittances and independent current sources. This approach will work well as long as every component can be modeled in the admittance matrix or can be converted into equivalent injection current.

The methods for three-phase distribution networks can be basically divided into two classes, Gauss-Seidel (Teng 2002; Vieira, Freitas et al. 2004) and Newton-Raphson (Garcia, Pereira et al. 2000; Lin and Teng 2000; da Costa, de Oliveira et al. 2007). Gauss-Seidel method needs much iteration and is known to be slow. Newton-Raphson has good convergence characteristic, but the Jacobian that needs to be partially or totally calculated in every iteration makes this approach unattractive.

This paper presents three-phase power flow for unbalance distribution system using forward-backward propagation technique. The algorithm works directly on the system without any modification. Therefore, there is no need to decompose the system into symmetrical components as well as to decouple the system into individual phases. However, the conversion of load and shunt element into their equivalent injection current is necessary. Distribution line charging is usually too small to be included (Lin and Teng 2000). The algorithm is implemented on IEEE 34-bus system including asymmetrical lines and unbalance loads.

System Modeling

Line Modeling

The accuracy of three-phase power flow results greatly depends on the line impedance model used. Therefore, an exact model of a three-phase line section needs to be firstly developed. The model of distribution line feeder as in (Kersting and Phillips 1995) will be developed and used in this paper. An equivalent circuit for a three-phase line section is shown in Fig. 1.

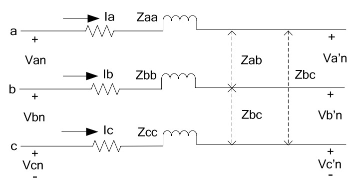

Fig. 1. Three-phase line model

The modeling of three-phase lines starts with determination of self and mutual impedances of a line section which are functions of the conductors and the spacing between conductors on the pole or underground. The "modified" form of Carson's equations are used to determine the self and mutual impedances of this model and are given by the following equations:

$$z_{ii} = r_i + 0.0953 + j0.12134 \times \left[\ln(1/GMR_i) + 7.934\right] \ \Omega/\text{mi} \qquad (1)$$

$$z_{ij} = 0.0953 + j0.12134 \times \left[\ln(1/D_{ij}) + 7.934\right] \Omega/\text{mi} \qquad (2)$$

Where r_i is the conductor resistance (Ω/mile), GMR_i is the conductor geometric mean radius (ft), and D_{ij} is the spacing between conductors i and j (ft). Application of (1) and (2) to the three-phase line indicated in Fig. 1. results in a 4×4 "primitive impedance matrix":

$$\left[Zprim\right] = \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} & z_{an} \\ z_{ba} & z_{bb} & z_{bc} & z_{bn} \\ z_{ca} & z_{cb} & z_{cc} & z_{cn} \\ z_{na} & z_{nb} & z_{nc} & z_{nn} \end{bmatrix} \qquad (3)$$

By applying Kron reduction, this matrix may be reduced into 3×3 "phase impedance matrix" whose elements are determined by the following equation:

$$Z_{ij} = z_{ij} - z_{in} z_{nj} / z_{nn} \qquad (4)$$

And the resulted "phase impedance matrix" is:

$$\left[Z_{abc}\right] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \qquad (5)$$

Load Modeling

The load (balance and unbalance) is represented by its equivalent injection current. The load modeling as in (Vieira, Freitas et al. 2004) is adopted in this paper. For three-phase loads connected in wye or single-phase loads connected line to neutral, the equivalent injection currents at k^{th} bus are determined by the following equation:

$$I_m^k = \frac{P_m^k - jQ_m^k}{V_m^k *}; \; m \in [a, b, c] : \text{phase} \tag{6}$$

Where P_m, Q_m, V_m^* denote real power, reactive power, and complex conjugate of the voltage phasor for each phase, respectively. For three-phase loads connected in delta or single-phase loads connected line to line, the equivalent injection currents at the k^{th} bus are determined by the following equations:

$$\begin{aligned} I_a^k &= \frac{P_{ab}^k - jQ_{ab}^k}{V_a^k * - V_b^k *} - \frac{P_{ca}^k - jQ_{ca}^k}{V_c^k * - V_a^k *} \\ I_b^k &= \frac{P_{bc}^k - jQ_{bc}^k}{V_b^k * - V_c^k *} - \frac{P_{ab}^k - jQ_{ab}^k}{V_a^k * - V_b^k *} \\ I_c^k &= \frac{P_{ca}^k - jQ_{ca}^k}{V_c^k * - V_a^k *} - \frac{P_{bc}^k - jQ_{bc}^k}{V_b^k * - V_c^k *} \end{aligned} \tag{7}$$

Where P_n, Q_n, $n \in [ab, bc, ca]$, represents the real and reactive load connected between the respective phases, and V_m^* , $m \in [a, b, c]$ denotes the complex conjugate of the voltage phasor for each phase, respectively.

Shunt Admittance Modeling

Three phase shunt capacitors can also be represented by equivalent injection currents (Vieira, Freitas et al. 2004). Assuming that the capacitor bank has an ungrounded wye connection, the current injection in each phase can be expressed by:

$$\begin{aligned} I_a^{sh} &= \frac{y^{sh}}{3}\left(-2V_a^k + V_b^k + V_c^k\right) \\ I_b^{sh} &= \frac{y^{sh}}{3}\left(V_a^k - 2V_b^k + V_c^k\right) \\ I_c^{sh} &= \frac{y^{sh}}{3}\left(V_a^k + V_b^k - 2V_c^k\right) \end{aligned} \tag{8}$$

Where $y^{sh} = jQ^0/|V^0|^2$; Q^0 is the nominal reactive power per phase and $|V^0|$ is the magnitude of the nominal voltage each phase. If the capacitor bank has a grounded wye connection, then the injection current is:

$$\begin{aligned} I_a^{sh} &= -y^{sh}V_a^k \\ I_b^{sh} &= -y^{sh}V_b^k \\ I_c^{sh} &= -y^{sh}V_c^k \end{aligned} \tag{9}$$

Forward-Backward Algorithm

Power flow is calculated by initially mapping the distribution network to determine forward and backward propagation paths followed by branch current calculation and bus voltages calculation.

Branch Current Calculation

Using backward propagation path, branch current can be calculated using the equivalent bus injection currents. The branch current is successively calculated for the network ends toward the source (swing). The voltage at each bus therefore needs to be firstly determined. For the first iteration, the voltage at each bus is set to 1.0 pu with the angle of 0, -120 and 120 degrees for phase a, b, and c, respectively. These voltages are updated during the iteration and therefore the injection currents will also change.

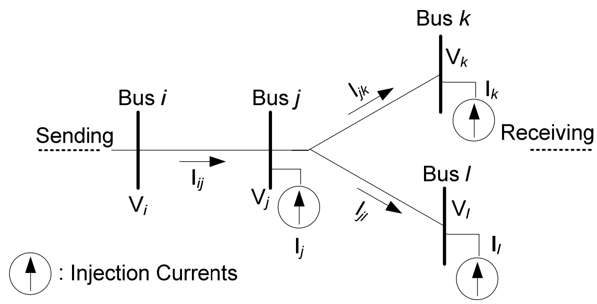

Fig. 2. Part of a distribution system

Fig. 2 indicates a part of distribution system and bus injection currents. The relationships between branch currents and injection currents are:

$$
\begin{aligned}
I_{jk} &= -I_k \\
I_{jl} &= -I_l \\
I_{ij} &= I_{jk} + I_{jl} - I_k
\end{aligned}
\tag{10}
$$

Where I_{jk} is the branch current between bus *j* and bus *k*, and I_j injection currents at bus, respectively.

Bus Voltage Calculation

Using forward propagation path, the voltage at each bus is calculated using the obtained branch currents and line impedance. The voltages are consecutively calculated from the source (swing) toward the network ends. The voltage of swing bus is kept constant at $1.0\angle 0^0$ pu, $1.0\angle\text{-}120^0$ pu, and $1.0\angle 120^0$ pu for phase a, b, and c, respectively. For the part of system indicated in Fig. 2, the bus voltages can be calculated as follows:

$$
\begin{aligned}
V_j &= V_i - Z_{ij} I_{ij} \\
V_k &= V_j - Z_{jk} I_{jk} \\
V_l &= V_j - Z_{jl} I_{jl}
\end{aligned}
\tag{11}
$$

Where V_j is the voltage at bus *j* and Z_{jk} is the impedance of line section between bus *j* and *k*. The updated bus voltages are then used to calculate the bus injection currents. It should be noted that all calculations are carried in the three-phase frame.

Convergence Criteria

The outlined steps for branch currents and bus voltages calculations are invoked during the power flow iteration. The iteration converges if the different of bus voltages for the consecutive iterations is equal to or less than the prescribed tolerance. The voltage mismatch of bus *j* at the n^{th} iteration is given by:

$$
\begin{aligned}
&\Delta V_j^n = V_j^n - V_j^{n-1} \text{ for phase a, b, c} \\
&\mathrm{Re}(\Delta V_j^n) < \varepsilon\,;\ j \in \text{all buses} \\
&\mathrm{Im}(\Delta V_j^n) < \varepsilon\,;\ j \in \text{all buses}
\end{aligned}
\tag{12}
$$

Where ε is the prescribed tolerance. If these equations are satisfied, then the iteration stops. Otherwise, iteration process is repeated. Once the load flow iteration converges all the branch currents and voltage at each bus are known. The real and reactive power loss can therefore be calculated. The three-phase power flow algorithm using Forward-Backward Propagation technique is given in the flowchart of Fig. 3.

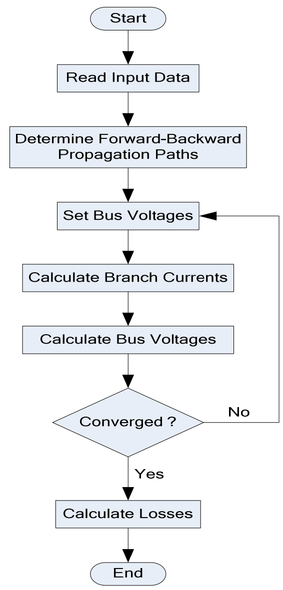

Fig. 3. The flowchart for three-phase power flow calculation using Forward-Backward Propagation technique

Simulation

System Data

The simulation is carried out for the IEEE 34-bus system (Kersting 1991) indicated in Fig. 4. The system includes balance as well as unbalance loads. A minor modification is performed for the system to only include three phase asymmetrical lines. However, the load data and the capacitor data are remain the same. The system load and branch data are respectively given in Table 1 (A and B) and Table 2.

Table 1(a) Balance Loads of the IEEE 34-bus System

Bus#	Y/Δ	Phase A		Phase B		Phase C	
		kW	kVAr	kW	kVAr	kW	kVAr
860	Y	19.91	15.94	19.91	15.94	19.91	15.94
840	Y	8.86	7.09	8.86	7.09	8.86	7.09
844	Δ	133.444	106.83	133.444	106.88	133.444	106.88
848	Y	19.45	15.57	19.45	15.57	19.45	15.57
890	Δ	27	21.62	27	21.62	27	21.62

Note

1. Swing: bus 800
2. MVA base: 2.5 MVA

Table 1(b) Unbalance Loads of the IEEE 34-bus System

Bus#	Y/Δ	Phase A		Phase B		Phase C	
		kW	kVAr	kW	kVAr	kW	kVAr
806	Y	0	0	31.22	16.14	26.07	13.84
810	Δ	0	0	15.88	8.21	0	0
820	Δ	33.9	17.52	0	0	0	0
822	Y	135.5	70.07	0	0	0	0
824	Y	0	0	0.39	0.2	0	0
826	Y	0	0	41.93	21.68	0	0
828	Δ	0	0	0	0	2.78	1.44
830	Δ	6.18	3.2	0	0	0	0
834	Y	3.99	2.06	12.55	6.49	12.82	6.63
836	Δ	27.37	14.15	10.55	5.45	42.05	21.74
838	Δ	27.61	14.27	0	0	0	0
840	Y	17.49	9.04	21.81	11.27	0	0
842	Y	0	0	0	0	0	0
844	Δ	9.12	4.71	0	0	0	0
846	Y	0	0	24.59	12.71	22.23	11.49
848	Y	0	0	22.62	11.7	0	0
856	Y	0	0	3.71	1.92	0	0
858	Δ	6.68	3.45	1.08	0.56	5.35	2.77
860	Y	15.66	8.09	20.86	10.78	111.15	57.46
862	Y	0	0	0	0	0	0
864	Δ	0.63	0.33	0	0	0	0

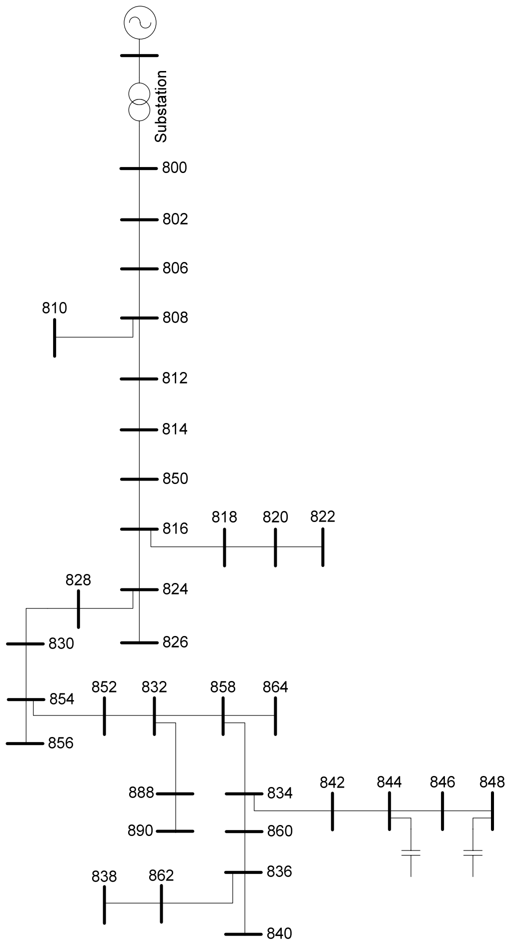

Fig. 4. The IEEE 34-bus system used for simulation

Table 2. Branch Data of The IEEE 34-Bus System

Bus		Config.	length	Line Condctr		Neutral Condctr	
Fr	To		(ft)	R	GMR	R	GMR
800	802	BACN	2580	1.69	0.00418	1.69	0.00418
802	806	BACN	1730	1.69	0.00418	1.69	0.00418
806	808	BACN	32230	1.69	0.00418	1.69	0.00418
808	810	BCAN	5840	2.55	0.00452	2.55	0.00452
808	812	BACN	37500	1.69	0.00418	1.69	0.00418
812	814	BACN	29730	1.69	0.00418	1.69	0.00418
814	850	BACN	10	1.69	0.00418	1.69	0.00418
816	818	ABCN	1710	2.55	0.00452	2.55	0.00452
816	824	BACN	10210	1.69	0.00418	1.69	0.00418
818	820	ABCN	48150	2.55	0.00452	2.55	0.00452
820	822	ABCN	13740	2.55	0.00452	2.55	0.00452
824	826	BCAN	3030	2.55	0.00452	2.55	0.00452
824	828	BACN	840	1.69	0.00418	1.69	0.00418
828	830	BACN	20440	1.69	0.00418	1.69	0.00418
830	854	BACN	520	1.69	0.00418	1.69	0.00418
832	858	BACN	4900	1.69	0.00418	1.69	0.00418
832	888	BACN	100	1.69	0.00418	1.69	0.00418
834	860	BACN	2020	1.69	0.00418	1.69	0.00418
834	842	BACN	280	1.69	0.00418	1.69	0.00418
836	840	BACN	860	1.69	0.00418	1.69	0.00418
836	862	BACN	280	1.69	0.00418	1.69	0.00418
842	844	BACN	1350	1.69	0.00418	1.69	0.00418
844	846	BACN	3640	1.69	0.00418	1.69	0.00418
846	848	BACN	530	1.69	0.00418	1.69	0.00418
850	816	BACN	310	1.69	0.00418	1.69	0.00418
852	832	BACN	10	1.69	0.00418	1.69	0.00418
854	856	BCAN	23330	2.55	0.00452	2.55	0.00452
854	852	BACN	36830	1.69	0.00418	1.69	0.00418
858	864	ABCN	1620	2.55	0.00452	2.55	0.00452
858	834	BACN	5830	1.69	0.00418	1.69	0.00418
860	836	BACN	2680	1.69	0.00418	1.69	0.00418
862	838	ACBN	4860	1.69	0.00418	1.69	0.00418
888	890	BACN	10560	1.12	0.00446	1.12	0.00446

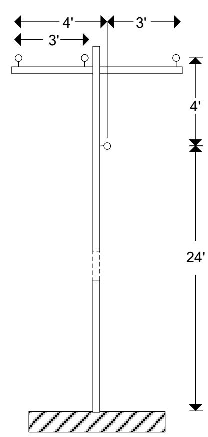

Fig. 5. Overhead line spacing for the IEEE 34-bus system

Fig. 5 shows the spacing distances between the phase conductors and the neutral conductor. The line configuration is given in Table 2 (3rd column) indicating that the phase conductors are sequentially placed starting from the left side position.

Simulation Results

The system is run using the algorithm presented in Fig. 3. The simulation is coded using Matlab version 7.0 (R14). For the system of Fig.4, the three-phase power flow calculation takes 5 iterations to converge. This iteration number is fairly small indicating that the algorithm is robust for unbalance power flow calculation. The results of simulation including magnitude and angle of voltage at every phase are given in Table 3. The simulation also indicates the real and reactive power losses of 8.2 kW and 3.4 kVAr, respectively.

Table 3. Simulation Results of The IEEE 34-Bus Unbalance System

bus	Volt at phase c		Volt at phase c		Volt at phase c	
no	mag	Angle	mag	Angle	mag	Angle
800	100	0.00	100	-120.00	100	120.00
802	99.93	-0.01	99.94	-120.01	99.94	119.99
806	99.88	-0.02	99.89	-120.02	99.90	119.98
808	99.01	-0.22	99.19	-120.17	99.17	119.80
810	99.01	-0.22	99.18	-120.18	99.17	119.80
812	97.99	-0.45	98.38	-120.35	98.35	119.58
814	97.18	-0.64	97.75	-120.49	97.70	119.41
816	97.17	-0.64	97.74	-120.49	97.69	119.41
818	97.14	-0.64	97.74	-120.49	97.69	119.41
820	96.32	-0.66	97.84	-120.49	97.66	119.49
822	96.11	-0.67	97.88	-120.49	97.65	119.52
824	97.02	-0.69	97.50	-120.54	97.47	119.33
826	97.02	-0.69	97.49	-120.54	97.48	119.33
828	97.01	-0.69	97.48	-120.54	97.46	119.32
830	96.72	-0.81	97.08	-120.63	97.01	119.17
832	96.20	-1.02	96.36	-120.79	96.19	118.88
834	96.07	-1.08	96.17	-120.84	95.97	118.80
836	96.05	-1.08	96.15	-120.84	95.94	118.80
838	96.04	-1.08	96.14	-120.84	95.94	118.80
840	96.05	-1.08	96.15	-120.84	95.94	118.80
842	96.07	-1.08	96.17	-120.85	95.97	118.80
844	96.07	-1.09	96.16	-120.86	95.96	118.79
846	96.08	-1.11	96.15	-120.87	95.97	118.77
848	96.08	-1.11	96.16	-120.87	95.97	118.77
850	97.18	-0.64	97.75	-120.49	97.70	119.41
852	96.20	-1.02	96.36	-120.79	96.19	118.89
854	96.71	-0.81	97.07	-120.63	97.00	119.16
856	96.71	-0.81	97.06	-120.63	97.00	119.16
858	96.14	-1.05	96.27	-120.82	96.09	118.85
860	96.06	-1.08	96.16	-120.84	95.95	118.80
862	96.05	-1.08	96.15	-120.84	95.94	118.80
864	96.14	-1.05	96.27	-120.82	96.09	118.85
888	96.20	-1.02	96.36	-120.79	96.19	118.89
890	96.18	-1.02	96.34	-120.79	96.17	118.89

Conclusion

Three-phase power flow for an unbalance distribution system is carried out using Forward-Backward Propagation Technique. The IEEE 34-bus unbalance system is used for simulation. The main conclusions are:

- The implemented algorithm works directly on the simulated system and, therefore, there is no need to transform the system into the symmetrical components as well as to decouple the three-phase into individual phases;
- The algorithm is robust for the radial unbalance distribution system with good convergence characteristic.

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