Curvature Mode Shape Changes to Detect Crack Damage in Non-Prismatic Reinforced Concrete Beams

Perubahan Bentuk Mode Kelengkungan Untuk Mendeteksi Kerusakan Retak Pada Balok Beton Bertulang Tak-Prismatis

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ABSTRACT

Non-prismatic beams are widely used in many engineering structures such as bridge girder and long-span structure. Damages in a reinforced concrete beam are categorized as light/small, medium, and wide cracks. Every typical crack-damage requires a special treatment to strengthen its structure. A reinforced concrete beam experiencingcrack damages reducesits stiffness and strength. This paper presents an investigation of non-prismatic reinforced concrete beams considering curvature mode shape changes to locate damage. Laboratory vibration tests were undertaken in an attempt to measure crack damage taking into account a modified flexure damage index (MFDI) expressed in terms of frequencies.Based on experimental studies and finite element modeling show that this approach is practicallyused for detecting crack damage in reinforced concrete beams, which exhibit high non-linearity behavior.
Keywords: Damage detection, crack, non-prismatic, reinforced concrete beam, finite element modeling

ABSTRAK

Balok dengan dimensi/inersia tidak merata sepanjang batang/tak-prismatik (non-prismatic) secara luas dipakai untuk berbagai rekayasa struktur, seperti pada pelagar jembatan dan struktur bentangan panjang. Kerusakan-kerusakan pada balok beton bertulang digolongkan sebagai kerusakan kecil, sedang, dan lebar. Setiap tipe kerusakan retak memerlukan perbaikan khusus untuk memperkuat strukturnya.Suatu balok beton bertulang yang mengalami kerusakan-kerusakan retak mengurangi kekakuan dan kekuatannya. Makalah ini mempresentasikan sebuah pengujian balok beton bertulang tak-prismatik dengan mempertimbangkan perubahan-perubahan bentuk mode kelengkungan untuk melokasikan kerusakan. Pengujian vibrasi di laboratorium dilakukan dengan maksud untuk mengukur kerusakan retak dengan memperhitungkan indek kerusakan lentur termodifikasi (MFDI) yang diekspresikan dalam frekuensi-frekuensi. Berdasarkan studi eksperimen dan pemodelan elemen hingga menunjukkan bahwa pendekatan ini secara praktis digunakan untuk mendeteksi kerusakan retak pada balok beton bertulang, yang menunjukkan perilaku tak-linier yang tinggi.
Kata-kata kunci: Deteksi kerusakan, retak, tak-prismatik, balok beton bertulang, pemodelan elemen hingga

INTRODUCTION

Dynamic system identification of reinforced concrete (RC) beams i.e. natural frequencies, damping ratios and mode shapes is of high importance due to their special roles in most civil engineering structures (Daneshjoo and Gharighoran, 2008).

Identification of dynamic characteristics of anystructural system is known as dynamic system identification.Crack damage is detected by comparing the identified dynamical indices of the damaged and undamaged structures. Due to the special role of the beamsin most of the civil engineering structures, dynamics system identification of beams, especially during service time, is of high importance. For example, incase of failure, the beams as an important element ofthe bridges may cause overall instability of thebridge structure. In this regard, various studies havebeen conducted. Monitoring of bridges and buildingbased on vibration measurements is widely addressedin literatures, e.g., in Doebling et. Al (1998) an extensiveoverview is given.

Finite Element Method (FEM) is a numerical approach of computing the real structure by considering several assumptions, which can lead to heavy differences betweenthe model and the real structure. Its result accuracy is dependent on several factors such as nonlinear material modeling, selected element model, and advanced numerical method used. Through the comparison of experimental data from dynamic testingand numerical data, these differences can be usually well highlighted.

One of the most promising computational methods in the field of Structural Health Monitoringis the Finite Element Model Updating (FEMU). The FEMU is anumerical technique used to minimize the differences between the real structure and the finite element model. Infact, the finite element (FE) model properties (e.g. mass, stiffness,boundary conditions) can be modified by the updating procedure whilst the resulting structure presents abetter dynamic agreement with the physical reality.Consequently, the FEMU can be employed also tothe purpose of damage detection. By utilizing experimental data from a damaged structure and applying the updating procedure, the resulting structural parameter distributions (e.g. elastic modulus) canpoint out a deficiency in the structural properties, which means the occurrence of damage. In this optic,the FEMU can be proficiently applied in the field ofStructural Health Monitoring.

It is easily accepted that when damage occurs, a structure would suffer a decrease in stiffness and as a consequence there is a decrease in natural frequencies of vibration. For a beam structure, a loss in stiffness would imply an increase in curvature of the elastic, which can be used for damage detection (Pandey et al, 1991, Wahab and Roeck, 1999). It also changes dynamic characteristics and this change has been exploited for the same purpose (Cawley and Adams, 1979, Spyrakos et al, 1990). This study aims at relating damage to the change in curvature and hence natural frequencies in a non-prismatic concrete beam when cracks develop in the process of loading.

THEORETICAL REVIEW

The formula giving angular natural frequencies of a prismatic beam of length L, simply supported with uniform cross-section is widely available in literature, using standard notations:

$$\omega_n = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EI}{m}} \tag{1}$$

where: n is the order of the mode shape, m is mass per unit length. Engineering beam theory gives:

$$\frac{1}{EI} = \frac{\kappa}{M} \tag{2}$$

where κ is the curvature of the beam's axis, M is the bending moment at the cross-section considered. Equations (1) and (2) give:

$$\kappa = \frac{n^4 \pi^4 M}{mL^4} \frac{1}{\omega_n^2} \tag{3}$$

Equation (3) is derived for a linear prismatic beam and is not exact for a non-linear non-prismatic beam. However it shows that there is a relationship between the curvature and the natural frequencies and this will also be true for non-linear non-prismatic beams. On the other hand curvature can always be found from the beam deflection by:

$$\kappa = \left[\frac{\partial^2 v}{\partial x^2}\right]\left[1 + \left(\frac{\partial^2 v}{\partial x^2}\right)^2\right]^{-3/2} \tag{4a}$$

and, in the limit of small deflections and slopes

$$\kappa = \frac{\partial^2 v}{\partial x^2} \tag{4b}$$

The last term in Equation (4b) can be approximated by using a central difference formula:

$$\kappa = \frac{v_{i+1} - 2v_i + v_{i-1}}{l^2} \tag{5}$$

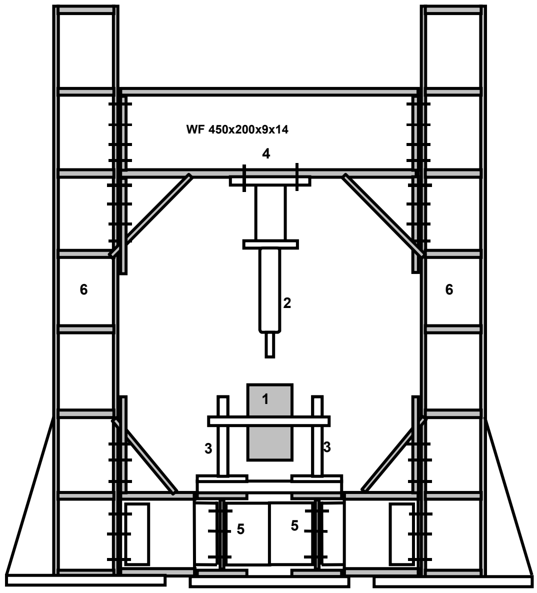
where l is the grid length of the measuring grid (or the element size of the finite element in a numerical solution) and $v_i$is the deflection of the beam at the cross-section considered. As a consequence, curvature mode shape can be obtained from displacement mode shape. When the beam is non-prismatic the finite element method (FEM) can be used for static and dynamic analysis. However when cracks exist the FEM modeling is not straight forward as the behavior of reinforced beam having cracks is far from linear, because of the non-linearity of concrete but also because of the complex interplay between reinforcing steel, concrete and the presence of the many cracks that exist in the concrete material due to the low tensile strength of concrete (Warner et al, 1998).

RESULTS AND DISCUSSION ON NON-PRISMATIC REINFORCED CONCRETE BEAM

When there is crack damage the natural frequencies would change as well as the curvature. In this study, it is proposed to relate the change in curvature with evidence of damage. The change in curvature is obtained from the displacement mode shape (displacement mode curve) of a finite element model (FEM) of the beam by using Equation (5) and cracks are detected by surface observation with a microscope. It is noted that with the support and loading arrangement the first cracks would appear in the midspan. As the load increases the cracking extends outwards from this point. The FEM model should incorporate the current damage inflicted on the beam and detected by the microscope. The natural frequencies can be either found by vibration testing or from modal analysis of the FEM model.

1. Cracking of the simply supported non-prismatic beam

Cracking is produced by a concentrated static load at the midspan, which is increased progressively and exerted by a hydraulic jack in the experimental set-up shown in Figure 1.



1. Concrete beam *4. Test jig portal frame*
2. Hydraulic jack *5. Test jig beams*
3. Support *6. Test jig column*

Figure 1. Experimental set up

In this study there were four typical beam specimens classified into two groups i.e. first group of beams *E* and *F,* and second group of beams *G* and *H*. The averaged concrete compressive strengths of 33.96 and 38.22 MPa were used in the first and second groups of specimen, respectively. All specimens were strengthened with four flexural reinforcements with 10 mm diameter and ligature pitches of 6 mm diameter were shown in Figure 2. The first group of specimen was treated anything, whilst in the second group of beams artificially used a peace of plastic applying in the tensile fiber of the concrete at the nodal point 15 as an assumption of initial crack.

The detail of a typical non-prismatic reinforced concrete beam is shown in Figure 2.The area in the midspan is monitored for cracks using a microscope. Load-displacement curves obtained in static tests for the midspan position for four beams labeled *E, F, G, H*, are shown in Figure 3. This Figure shows the non-linear behavior and, also, the scatter of results in nominally identical beams. Particular attention was taken to determine the load P_{cr} when first crack appears.

A maximum load P_{max} is assigned when displacement increases excessively fast while load practically does not increase, the beam is considered 100% damage. For intermediate loads between P_{cr} and P_{max}, a damage index is defined as:

$$DI = \frac{P - P_{cr}}{P_{max -} P_{cr}} \tag{6}$$

This definition of *DI* is widely accepted but is arbitrary as P_{cr} relates to the onset of observable crack when some damage may have been already inflicted.

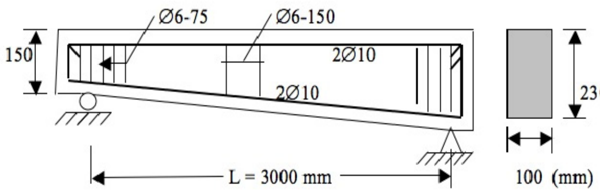


Figure 2. A typical non-prismatic reinforced concrete beam

Figure 3 presents the relationships between load and displacement responses for all beam specimens tested with an experimental set up shown in Figure 4. The first group of two beams (*E* and *F*) depicts a similar trend to the strength resulting very close displacement values but it differently reaches the maximum load. Another group of the beam significantly shows different strength performance. The beam *G* has better performance in resisting the displacement response, whilst the beam *H*approximately fails in about 50% of the maximum displacement value. It can be generally concluded that the beam *G* has better displacement ductility compared to other beams, however the intact beams present better strengths providing higher achievement of applied loads. In contrast, the beam *H*provides smallest displacement ductility as well as its strength.

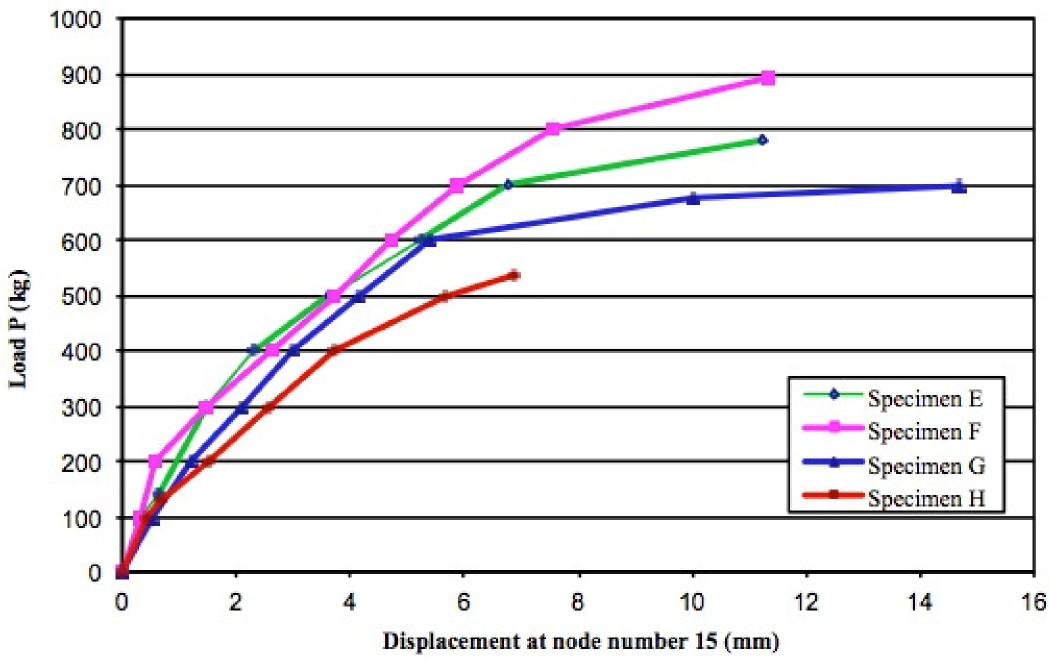


Figure 3. Load – displacement curve

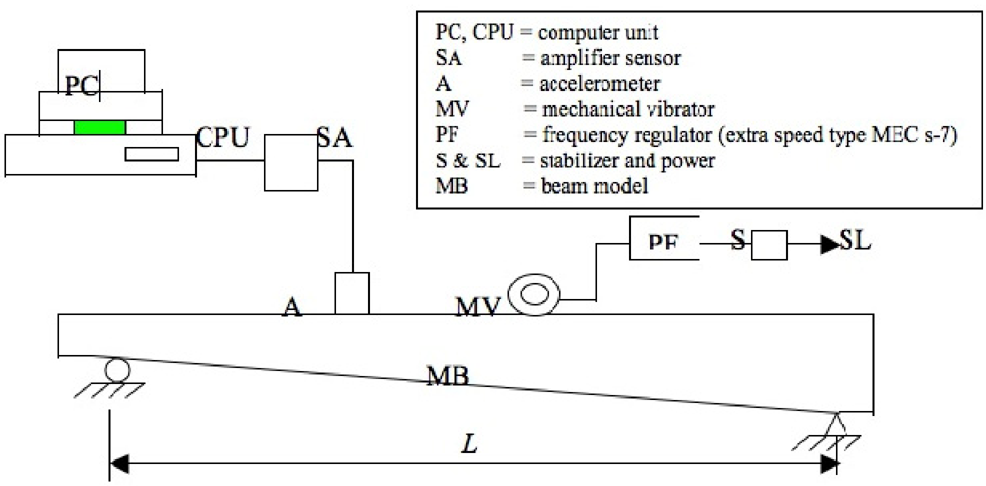


Figure 4. Experimental set up (PC: Computer and Data Acquisition; SA: Sensor amplifier; A: accelerometer; MV: mechanical vibrator; F: Frequency generator and power source)

2. Determination of Natural Frequencies by Vibration Racking of the Simply Supported Prismatic Beam

For a beam, undamaged or damaged, its natural frequencies can be determined by vibration test using mechanical vibrator employing eccentric masses. Accelerations recorded by an accelerometer at the midspan and another moving transducer were acquired and fed into computer to determine the natural frequency and displacement mode shape. The experimental set up is shown in Figure 4.

Equation (3) shows that for the damaged beam, the change in ω_n is intimately related to κ. This is the basis of defining a modified flexure damage index to indicate the extent of damage (Dipasquale and Akhmet, 1987):

$$MFDI = \frac{(1/\omega^2) - (1/\omega^2_{cr})}{(1/\omega^2_{\max} - 1/\omega^2_{cr})} \quad (7)$$

where ω = natural frequency of the damaged beam, ω_{cr} = natural frequency at the first crack, and ω_{max} = natural frequency at the maximum crack. Typical experimental results are shown in Table 1 for beam *G*, other beams gave similar results (Saleh, 2000).

Table 1. Experimental results for beam *G*

Load (kg)	Natural Frequency (Hz)	Note	DI	MFDI
0	32.227	No Damage		
100	32.227	No Damage		
200	31.900	First Crack	0.000	0.000
300	30.355		0.200	0.399
400	29.297		0.400	0.709
500	29.053		0.600	0.786
600	28.869		0.800	0.845
673	28.527		0.946	0.957
700	28.400	Maximum load	1.000	1.000

It should be noted that the static load was used to induce cracks but was withheld for the vibration test to find the natural frequency and displacement mode shape. It can be seen that the natural frequency remains independent of the static load applied when there are no cracks.

3. Finite Element Modeling

The presence of damage or deterioration in a beam structure causes changes in the natural frequencies of the structure. The most useful damage location methods (based on dynamic testing) are probably those using changes in resonant frequencies because frequency measurements can be quickly conducted and are often reliable. Abnormal loss of stiffness is inferred when measured natural frequencies are substantially lower than expected. Frequencies higher than expected are indicative of supports stiffer than expected. It would be necessary for a natural frequency to change by about 5% for damage to be detected with confidence. However, significant frequency changes alone do not automatically imply the existence of damage since frequency shifts (exceeding 5%) due to changes in ambient conditions have been measured for both concrete and steel bridges within a single day (Hong-ping et al. 2005).

In the FEM model, the beam of 3 m is divided into 30 elements using beam element of SAP2000 (Wilson, 2000). It is proposed to represent the damage inflicted to an element as a reduction in EI. This very crude FEM model is very simple but it avoids the intractable problem of modelling so numerous the number of cracks that exist in concrete. How much the reduction should be is not certain from the surface observation not to mention the complex behaviour of composite nature of concrete and reinforcing. The scheme adopted here is to compare FEM with experimental results for the undamaged beam first. Then for the damaged beam, a damaged element is modelled by reducing the value of the *EI* value in steps of 0.2 *EI* from *EI* to 0.2*EI* to search for a suitable reduction in *EI*. A modal analysis is then carried out for each scenario to get the mode shape and natural frequency. These are shown in Table 2. From each displacement mode shape, the corresponding curvature mode shape was obtained by using Equation (5).

Table 2. Natural frequencies from FEM with different values of *EI* for damaged elements (Beam *G*)

Percentage of EI	Loaded to 200 kg	Loaded to 400 kg	Loaded to 700 kg
100	32.09	32.09	32.09
80	31.83	29.34	28.99
60	31.41	26.00	25.37
40	30.62	21.75	20.93
20	28.56	15.77	14.95
Estimated percentage of EI for same MFDI	85.19	79.71	73.67

The last row of Table 2 shows the estimation of residual strength in terms of percentage of *EI* that should have been used to assign to damaged elements in the FEM model to get the same natural frequency as the experimental value, hence the same *MFDI*. It can be seen that the equivalent *EI* of the damaged elements decreases with increasing load as expected, however at the maximum load there is still considerable strength left. A typical fundamental displacement mode shape obtained by experiments and FEA are shown in Figure 5 for Beam *G*.

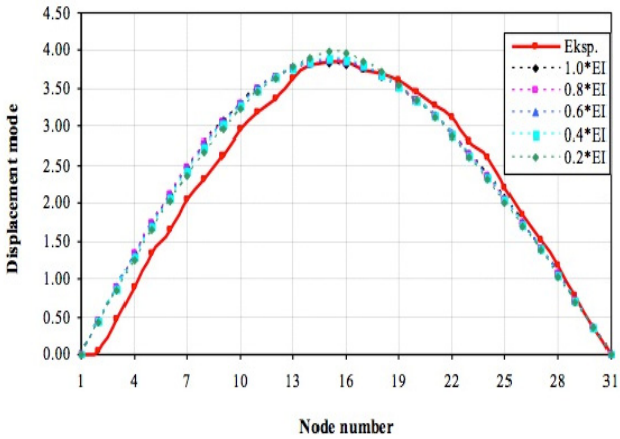
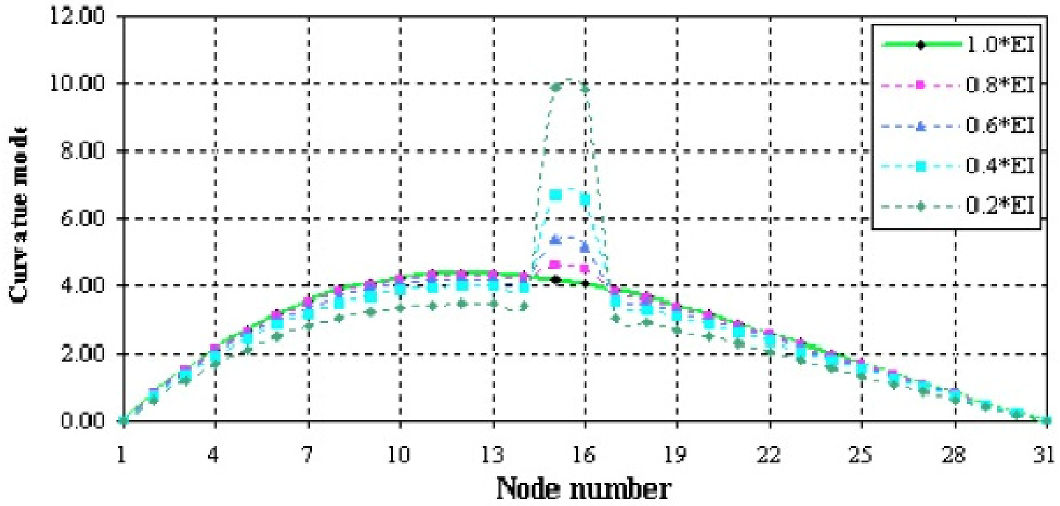


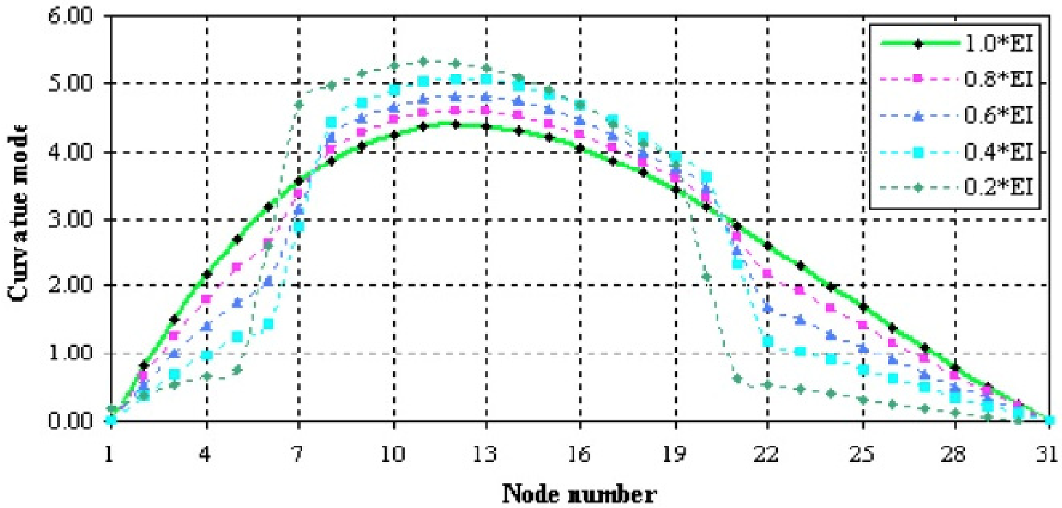
Figure 5. Displacement mode shape by experiment (solid line) and FEM results

A more interesting comparison is whether the curvature difference between the damaged beam and undamaged beam would help to identify the extent of damaged. Results are shown in Figure 6.

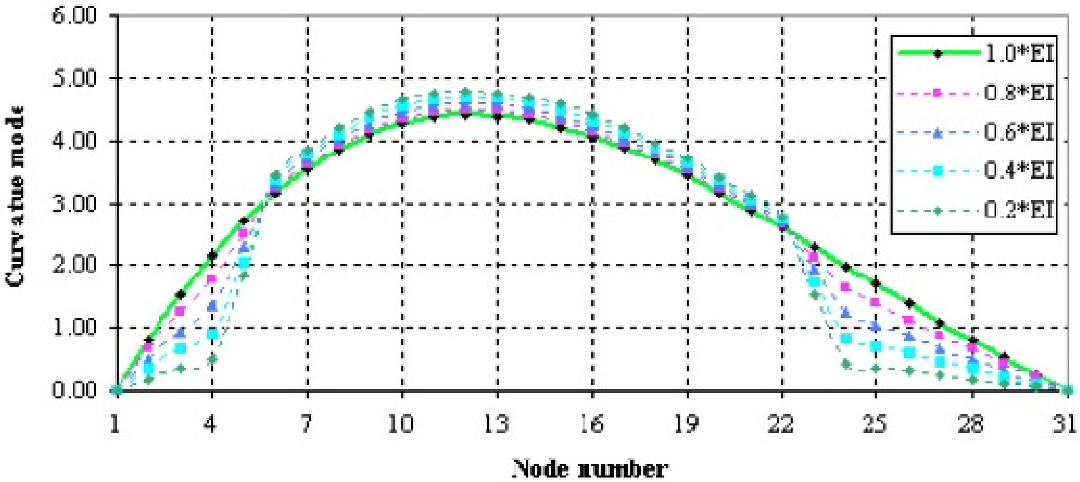
Inspection of displacement mode shapes in Figure 5 for the full range of reduced *EI* shows that displacement mode shapes are very similar and exhibit only small differences, they cannot reveal the location and extent of damage. However in Figure 6, it can be seen that with a wide range of projected values of reduced *EI* for damaged elements, all curvature mode shape curves show clearly a sudden change of curvature precisely at positions where damaged elements are identified by microscopic observation of cracks. This fact clearly shows that curvature mode shape can be used to indicate damage.



a) P/Pmax = 0.286



b) P/Pmax = 0.571



c) P/Pmax = 1.00

Figure 6. Curvature mode shapes for the specimen beam G

4. Displacement Mode and Natural Frequency Degradation

The static load was applied increasingly, starting from zero to reach first crack at the specimen. The load resulting the fracture on the specimen was marked with fracture load or crack load (P_r). In between the first and maximum loads, the median was determined or referred to as load of the second crack. The maximum static load (*Pmax*) was done with the same fracture load tests. It can be summarized on fracture load tests that crack natural frequency and maximum static load of each specimen are shown in Table 3. Based on these data, the non-dimensional of *f/fmax* and *P/Pmax* can be easily obtained and plotted ina graphat each specimen.

Table 3. Crack loading, crack natural frequency, and maximum static load.

No	Beam Type	P_r (N)	F_r (Hz)	P_{max} (N)
1	*E*	1392.594	32.959	7649.460
2	*F*	2942.100	33.691	8757.651
3	*G*	1961.400	31.900	6600.111
4	*H*	1274.910	32.959	5227.131

The specimen model was vibrated in the natural frequency with the impact excitation test, which recorded three times. In this case, the beam was manually hit by hand in any place along the beam surface with the natural frequencies averaged. For example the resulted natural frequency was 34 Hz, the lowest natural frequency of the model was 32.227 Hz; thereby the displacement mode was always lesser than the natural frequency of the lowest mode (model 1). This was observed from the relatively simple mode shape.

Although there were differences between the experimental and the numerical displacement values, the two modes still had similar patterns;

a) the maximum amplitude was occurred at point 15 and their curves had a similarity,
b) at point 15, the maximum amplitude of the damaged beam was higher compared to the intact beam amplitude,
c) the displacement shape was directly correlated to the non-prismatic element stiffness.

The amplitude differences occurred in this research was due to the followings;

a) the instrument support base was probably not installed properly therefore the observed amplitude was not the actual amplitude of the specimen due to mixed vibration or additional support base vibration,
b) the measurement at each nodal was not conducted at the same time.

The test results showed the natural frequency decreasing along with the damage level increase. The results have shown a good agreement with the theory of the natural frequency, which was in proportion with the specimen stiffness by neglecting the mass difference. The natural frequency test was conducted three times at each step of static loading.

Figure 6 shows that the greater static load applied, the lower the natural frequency of the model. At small load applied, the frequency decrease tends to linear curve, while at higher static load the curve decreases gradually and nonlinear curve. This result was consistent with a research undertaken by Spyrakos et al. (1990) expressing that the natural frequency of the beam reduced as well as the damage level propagation.

At both beams *G* and *H* had been applied with vertical an artificial crack or initial crack/damage by allocating a peace of thin plastic at point 16 in the tensile fiber. It shows the crack and maximum loads resisted by beams*G* and *H* were smaller compared to the intact beams (*E* and *F*). Referring to the compressive strength of concrete, it correlates directly in resisting the static loads. Instead of the compressive strength of concrete, as well as the ratio of *P/Pmax* increases the ratio of frequency *f/fmax* at first fracture decreases. Though beam *E* was not applied the initial crack, however due to the lower compressive strength of concrete used in this beam the ratio of *P/Pmax*was smallest.

CONCLUSION

In this contribution, the application of Finite Element Model Updating to the damage detection onbeam structures is presented. The updating proceduresare implemented in SAP2000. The code is here applied to damage detection on reinforced concrete beam structures.

At first, a simply supported beam with simulated damage is investigated. This means that the experimentaleigendata are numerically generated by SAP2000. By using frequencies and mode shape changes, itis possible to effectively locate and quantify the damage with high accuracy.

Subsequently, the real case of a reinforced concrete beam is examined. This structure was artificially damaged in a laboratory test inducing an asymmetrical damage distribution. By combining frequenciesand mode shape changes, it is possible to accurately locateand quantify the damage. When possible, the results were compared with outcomes from other studies.

When there are cracks in a beam, the change in natural frequency can only be used as a qualitative indication of damage. It was shown that the change in displacement mode shape is too small to be useful. However, for a damaged beam, the change in frequency is intimately related to the change in curvature. The curvature mode shapes exemplify the change due to damage and can be used to locate the extent of damage and location of damage, justifying the use of Modified Flexure Damage Index (*MFDI*) as an indication of residual strength. Curvature mode shapes are however very tedious to obtain experimentally. They can be obtained by FEM but further work is needed to improve the modeling of damage due to cracks in concrete in a simple yet effective way for the prediction of curvature mode shape. On the other hand, *MFDI* is entirely expressed in frequency terms, which can be obtained by vibration testing, is simple and realistic measure of the damage inflicted by cracks in non-prismatic reinforced concrete beams widely used in civil engineering structures.

Concrete is essentially facture material and therefore the direct detection on the damage location could only be identified depending on its micro cracks spreading out around the surface area along the tensile fiber. The external fracture/crack on the surface of the beam was read using the micro crack instrument. In this research, the damage location of the beam was observed by direct visual observation instead of estimating the change of the curvature mode shape which resulting the discontinuous curve at each load step. Based on this curve, the damage location can be identified due to the concentrated load acting at the middle of the beam.

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