BACKORDER AND LOST SALES CONTINUOUS REVIEW RAW MATERIAL INVENTORY CONTROL SYSTEM WITH LEAD TIME AND ORDERING COST REDUCTION

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Abstrak: The purpose of this research is to suggest the optimal raw material inventory system alternatives synchronized with the stock out characteristics and the condition can be occurred, the backorder raw material inventory control, beside that it also can reduce lead time and raw material ordering cost. In this case, the inventory models also extent (Q, R) Inventory Model Under lead Time and Ordering Cost Reduction with lead time and ordering cost can be reduced. After the calculation, the optimal solution of inventory models can be obtained; those are with backorder condition which produced the annual inventory total cost for the company.

Keywords: inventory, cost reduction, lead time, backorder, lot sales

INTRODUCTION

Extend (Q, R) Inventory Model Under Lead Time and Ordering Cost Reduction by Kun Shan Wan & I Chuan Lin (2004) [6] is a probabilistic model of Continuous Inventory Review (Q, R) including variable of lead time which can be used in Backorder and lost sales conditions. The main purpose of using this model is to having the optimal solutions simultaneously, including the amount of the economic order, decrease the order costs, reordering point and also decreased lead times, and minimize total inventory costs.

Inventory model is very important because there are many benefits come from the application of this model. When the company's applying the JIT (Just In Time) concept is very precise and give priority for the short lead times in delivery of raw materials or the production process schedule. Shorten the lead time is become the most effective and efficient way to get the application of JIT goals.

Meanwhile, in a real situation, the order costs of or preparation costs can be controlled and reduced with a lot of ways such as; work training, procedure changes, and specialized equipment combining. For example, the implementation of EDI (Electronic Data Interchange) will reduce costs and make the new reordering decision. Based on the successful Japanese experience in JIT program, the benefits come from by the order costs will be decrease in easily.

The main inventory purpose of the JIT program is producing small lot sizes in good quality. In order to achieve this goal, in reducing the order costs or the preparation costs in capital invested is one of the most effective ways. Moreover, produce the smaller lot sizes because the order cost or preparation costs lower, there

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are many advantages on it, such as; scheduling flexibility, smaller warehouse, and lower investment inventory.

Based on the industry problems, then the final task of this research aims to provide some alternative suggestions inventory control optimal system for the classification of raw material. This adapted to the characteristics and existing conditions in stock out which is, the Backorder and lost sales condition and can reduce lead time and cost of raw material delivery also.

THEORITICAL BACKGROUND

The inventory of raw materials divide in 2 alternative, first by using the model of Continuous Review (Q, R), and the second with the Extend model (Q, R).
Step to using this model found in Figure 1.

1. Getting past demand data and then used as input models. Past demand data that has nature probabilistic (uncertain).
2. Calculation process is performed by processing data requests and data inventory costs.
3. From the calculation process can be generated Q, R, SS, B, and the optimal total cost of supplies each year. Extend model (Q, R) can be optimal produced by S and L.
4. After calculating, have the production resumed.
5. As a feedback from production to meet demand, and then the expected demand data will be fulfilled and the possibility of stock out can be minimized.

Notation and Assumptions

A very important assumption in this model is the lead time which is comprised of several components, namely ordering inventory, transit reservations, and lead times from suppliers, delivery time and set up time. In practice, the lead time can be reduced by adding additional costs in other words lead time can be controlled. By reducing the lead time we can reduce safety stock, eliminating loss of profit due to stock out, improve customer satisfaction levels and also increase the competitive advantage in a business environment. The model of this inventory is certainly involving shortages or stock out condition is good in a Backorder or lost sales.

Explanation and assumptions developed assumptions in this model can be described as follows:

Notation

Q = number of orders (decision variable)
S = ordering costs / order (decision variable)
R = re-order point (decision variable)
L = length of lead time (decision variable)
D = average demand /year
h = cost savings / unit / year
π = stock out costs, backorder, or lost sales / kg
π_0 = profit / kg
β = possibility of stock out, backorder, or lost sales, $0 \le \beta \le 1$
$\bar{\pi}$ $= \pi + (1-\beta)\pi_0$
S_0 = first order cost
I(S) = large financial investment required to achieve cost S, $0 < S \le S_0$

θ = opportunity cost of investment / unit time
δ = percentage of messages in which the cost of S / Rp. decreased at the time of investment I(S)Increased
$\bar{x}_L$ = demand during lead time
σ_L = standard deviation of demand during lead time
X = demand during lead time with the average $\bar{x}_L$ And standard deviation $\sigma\sqrt{L}$, when σ is standard deviation demand per unit of time
Y = number of items received, is a random variable
$\psi(k)$ = $\{\phi(k) - k[1-\Phi(k)]\} = g(k)$
(from the standard normal table) ϕ,
Φ = standar normal probability density function (p.d.f) and d.f (from the standard normal table)
α = bias factor ($0 \le \alpha \le 1$) when the number of items received less than or equal with the number of orders Q

Assumptions

1. Reordering point R = demand during lead time + *safety stock* (SS), dan SS = k x (standard deviation of demand during lead time), can be explained, R = $\bar{x}_L + k\sigma_L$, where k is the safety factor.
2. Inventory is continuously reviewed and re-ordering occurs when inventory levels fall just at the point position reorders R.
3. Lead time L has n free components. Components - i have a minimum duration a_i and normal duration b_i, crashing cost per unit time c_i. To simplify, c_i is assembled as $c_1 \le c_2 \le ... \le c_n$. It is very clear that the decrease in lead time was first performed on component 1 (for having a crashing cost the most minimum), and continued component 2, and so on.
4. If we set $L_0 = \sum_{j=1}^{n} b_j$ and L_i i as a long lead time with components 1, 2, ..., i, i is changed to its minimum duration, then L_i can be expressed as $L_i = \sum_{j=1}^{n} b_j - \sum_{j=1}^{i} (b_j - a_j)$, i = 1, 2, ..., n ; and therefore the lead time crashing cost R (L) per cycle for $L \in [L_i, L_{i-1}]$, which can be explained by the following equation:
$$R(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$$
5. It is assumed that the investment of money, I (S), which serves to reduce the cost of messages is logarithmic function of the cost of the message S. This can be explained in the following equation: $I(S) = b \ln\left(\frac{S_0}{S}\right)$ with $0 < S \le S_0$, where $b = \frac{1}{\delta}$.
6. Variance of the number of items received are: $Var(Y \mid Q) = \sigma_0^2 + \sigma_1^2 Q^2$. In this study, it is assumed $\sigma_0^2 = \sigma_1^2 = 0$ and $\alpha = 1$, This shows that the number of items received equal to the number of orders (Q).

Basic model

Within this model, it can be determined: the cost of S is a variable decision and the decision sought to minimize the cost of investment capital to reduce the order cost of S and inventory costs by optimizing the Q, S, R, and L with limits $0 < S \le S_0$.

Therefore, the purpose of the problems mentioned above is to minimize the total cost of this inventory as mentioned below:

EAC(Q,S,R,L) = $\theta I(S) + EAC(Q,R,L)$

$$= \theta b \ln\left(\frac{S_0}{S}\right) + \frac{SD}{\alpha Q} + h\left[R - \bar{x}_L + (1-\beta)E(X-R)^+\right] + \frac{h}{2\alpha Q}\left[\sigma_0^2 + (\sigma_1^2 + \alpha^2)Q^2\right] + \frac{\bar{\pi}D}{\alpha Q}E(X-R)^+$$

$$+ \frac{R(L)D}{\alpha Q}, \quad \text{for } 0 < S \le S_0. \qquad \dots (1)$$

It is assumed that demand during lead time X follows a normal distribution with mean - average $\bar{x}_L$ and standard deviation $\sigma\sqrt{L}$. The equation is R = $\bar{x}_L + k\sigma_L$ and

$$E(X-R)^+ = \int_R^\infty (x-R)dF(x) = \int_k^\infty \sigma\sqrt{L}(z-k)d\Phi(z)$$

$$= \sigma\sqrt{L}\,\{\phi(k) - k[1-\Phi(k)]\}$$

$$= \sigma\sqrt{L}\psi(k)$$

$$= \sigma\sqrt{L}g(k),$$

when ϕ and Φ is the standard normal probability density function (pdf) and df, therefore we can determine the safety factor k as a decision variable in addition to reordering point R. Thus, the equation that can explain all these issues, for backorder conditions are:

$$\min EAC(Q,S,R,L) = \theta b \ln\left(\frac{S_0}{S}\right) + \frac{SD}{\alpha Q} + [k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\psi(k)]$$

$$+ \frac{h}{2\alpha Q}\left[\sigma_0^2 + (\sigma_1^2 + \alpha^2)Q^2\right] + \frac{\bar{\pi}D}{\alpha Q}\sigma\sqrt{L}\psi(k)$$

$$+ \frac{R(L)D}{\alpha Q}, \quad \text{for } 0 < S \le S_0. \qquad \dots (2)$$

To determine Q, we first determine

$$\frac{\partial EAC(Q,S,k,L)}{\partial Q} = -\frac{SD}{\alpha Q^2} - \frac{h\sigma_0^2}{2\alpha Q^2} + \frac{h}{2\alpha}(\sigma_1^2 + \alpha^2) - \frac{\bar{\pi}D}{\alpha Q^2}\sigma\sqrt{L}\psi(k) - \frac{R(L)D}{\alpha Q^2}$$

(2-30)

$$\frac{\partial EAC(Q,S,k,L)}{\partial S} = \frac{-\theta b}{S} + \frac{D}{\alpha Q} \qquad \dots (3)$$

$$\frac{\partial EAC(Q,S,k,L)}{\partial k} = h\sigma\sqrt{L} - h(1-\beta)\sigma\sqrt{L}(1-\Phi(k)) - \frac{\bar{\pi}D}{\alpha Q}\sigma\sqrt{L}(1-\Phi(k)) \qquad \dots (4)$$

$$\frac{\partial EAC(Q,S,k,L)}{\partial L} = \frac{1}{2}hk\sigma \quad L^{-1/2} + \frac{1}{2}h(1-\beta)\sigma L^{-1/2}\psi(k) + \frac{1}{2}\frac{\bar{\pi}D}{\alpha Q}\sigma L^{-1/2}\psi(k) - \frac{D}{\alpha Q}c_i \qquad \dots (5)$$

For the final solution of the Q, S, and k, the expected total cost of inventory each year will be happen in final position of the interval [L_i, L_{i-1}]. On the other hand, for a given value of $L \in [L_i, L_{i-1}]$, by determining the equation (3), (4), and (5) is zero, it can be determined:

$$Q = \sqrt{\frac{2D\left[S + \frac{h}{2D}\sigma_0^2 + \bar{\pi}\sigma\sqrt{L}\psi(k) + R(L)\right]}{h(\sigma_1^2 + \alpha^2)}} \qquad \dots (6)$$

$$S = \frac{\alpha\theta bQ}{D} \qquad \dots (7)$$

and

$$\Phi(k) = 1 - \frac{h\alpha Q}{h(1-\beta)\alpha Q + D\bar{\pi}} \qquad \dots (8)$$

RESEARCH METHODOLOGY

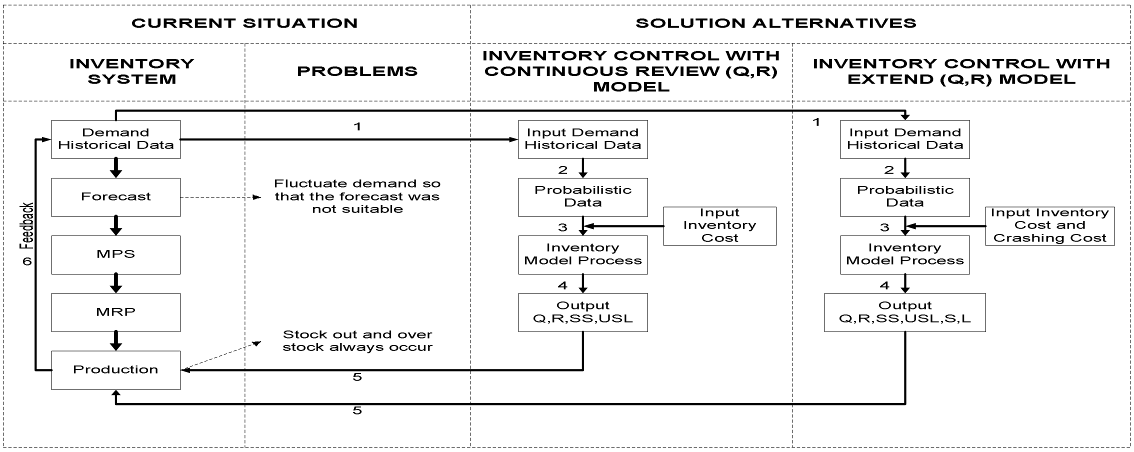

Figure 1. Framework Chart Research

Description Figure 1:
Q = Number Booking Economic
R = Point Reservations Back
SS = *Safety Stock*
USL = *Service Level*
S = Order Costs
L = *Lead Time*

Figure 1. The framework shows that inventory system used by companies today compared with the proposed system of raw material inventory control.

The company currently uses the MRP system (Material Requirement Planning) as a first step before the production activities. The process of the raw materials procurement are: the past demand data obtained, and then made forecasting, after it makes the MPS (Master Production Schedule), then of the MPS which made the production activity can be done.

Problems that arise are the past data does not make a specific pattern because fluctuate demand, so that forecasting is does not match with the actual demand

The next problem is the frequent occurrence of stock out and Overstock caused by fluctuate demand. To fix these problems in this research will be proposed control system

Probabilistic Continuous Review (Q, R) Inventory Model

Probabilistic inventory model used in this study is the Continuous Review inventory model. If we compared with the Periodic Review, this model has advantages including the Continuous Review model is very responsive when there is a shortage of goods (stock out), because it will soon make an order if the reorder point (reorder point) in reached while the model Periodic Review is unresponsive for the need to wait until the order arrives .

Continuous Inventory Model Review (Q, R) is used to calculate the optimal order number of Q while it will made on the amount of inventory reorder point R and useful to dampen fluctuations in demand. The calculation is happen to get the size step until we found the optimal size of Q and R are and the method used to obtain the optimal Q and R is the Joint Determination method of Q and R. . The process of calculation by this method consists of several iterations, where iteration is stopped if the Q values found and convergent of safety factor k. Backorder situation was occur in this company.

RESULT AND DISCUSSION

Calculation Process of Continuous Review Inventory Model (Q, R) Backorder Conditions

Continuous Review Model (Q, R) Backorder is an inventory model used when the company has a stock out of consumer's demand, so they do the fulfillment delay of the demand and immediately made an emergency order, the other hand, and consumers still waiting until the goods are available.

Percentage Decrease for Raw Materials that R Value exceed Q

The results that there are material items that have an R value exceeding Q. To overcome this distortion the R value would be reduced by lowering the percentage of the USL to obtain R values of less than Q.

The formula used in this calculation for the – j raw materials is:

1. Unit Stockout Rate :

   $USOR_j = 1 - USL_j$
2. Partial Expectation :

   $$g(k) = \frac{Q_j USOR_j}{\sigma_{Lj}}$$
3. Reorder Point :

   $$Rj = \bar{x}Lj + SSj = \bar{x}Lj + k\,\sigma_{Lj}$$

While other variables such as the calculation of R, B, SS, M, T, USL, and TC using the same formula as that used in Continuous Review (Q, R) Backorder Inventory Models.

Calculation Process for Extend Inventory (Q, R) Backorder Conditions

Iteration algorithm

Step 1. For every L_i, i = 0,1,2,...,n, do (i)-(v).

(i) Beginning with $S_{i1}=S_0$ and $k_{i1}=0$ (resulting $\psi(k)_{i1} = 0,3989$, which can be determined by checking the normal table $\phi(k_{i1}) = 0,3989$ and $\Phi(k_{i1}) = 0,5$).

(ii) Substitution S i1 and $\psi(k)_{i1}$ to equation (3), calculate the Q_{i1}.

(iii) By using Q i1 S i2 set of equations (4).
By using Q i1 specify $\Phi(k_{i2})$ from equation (5),

(iv) Then found by examining the table, and also obtained $\psi(k)_{i2}$.

(v) Repeat (ii) through (iv) until no change in the value of Q i, S i, and k i.

Step 2. Comparing S i and S 0.

(i) If S i <S 0, then the solution has been found in step 1 were optimal for given L i. Determine the solution by (Q_i^*, S_i^*, k_i^*).

(ii) If $S_i \geq S_0$, then for L_i, use the $S_i^* = S_0$, and (Q_i^*, k_i^*) can be determined by substitution S_i^* to equation (5), and solve (3) and (5) by iteration until there is convergent (the same solution obtained by Step 1).

Step 3. For each (Q_i^*, S_i^*, k_i^*, L_i), i = 0, 1, 2, ..., n, calculate the total cost of inventory every year is expected EAC(Q_i^*, S_i^*, k_i^*, L_i) using equation (1) for Backorder conditions.

Step 4. Discover $\min_{i=0,1,2,...,n}$ EAC(Q_i^*, S_i^*, k_i^*, L_i). If EAC(Q_N, S_N, k_N, L_N) = $\min_{i=0,1,2,...,n}$ EAC(Q_i^*, S_i^*, k_i^*, L_i), then (Q_N, S_N, k_N, L_N) is the most optimal solution. Please note, if k N and L N has been determined, then the reordering point $R_N = \bar{x}_{LN} + k_N \sigma_{LN}$ will follow. Then we find the Continuous Review (Q, R) Inventory Model Backorder Conditions Result Calculations for raw materials as results.

Step 5. The next step we find Continuous Review (Q, R) Inventory Model Backorder Result Calculation after the Reduction of USL (Results Rounding)
Total Lot = QJ before rounding / Minimum Lot Size , the results in-Roundup

Step 7_2 : Using the results of calculations that have been Q_{i1} obtained above, determine $OSOR_{i2}$ from equation OSOR, and then determine k_{i2} and $g(k_{i2})$ obtained from standard normal tables.

Iteration 3:

The calculating process on the iteration 3 and the next iteration using the step - the same steps listed at iteration 2 to get the optimal results when there was no change in the value of Qi, Si, and ki.

Step 6_3 : Based on calculations the Q_{i2} above, set S_{I3} using the equation for S.

Step 7_3 : By using the calculated Q_{i2} has acquired the above, determine $OSOR_{I3}$ of OSOR equation, and then determine k_{I3} and $g(k_{I3})$ is obtained from standard normal tables.

Step 9: Once booking result optimal number Q, the optimal message cost of S, and the safety factor k is optimal for each - each lead time, and β, then at this stage of the calculation process is continued by calculating R reordering point for each - each lead time and β.

Step 10 : The final stage in the process of this calculation is to calculate the total cost of inventory every year is expected to EAC (Q i, S i, k i, Li) for each lead time and β.

Extend (Q, R) Inventory Model Backorder Conditions Result Calculation Summary

The conclusion from the results of Extend (Q, R) Inventory Model Backorder conditions for raw materials we found the optimal solution (Backorder) β and S, , the amount of orders Q, reorder point R, and the total cost of inventory every year is expected EAC (Q, S, k, L).

Extend (Q, R) Inventory Model Backorder conditions for the classification of a raw material can be seen, Q has experienced rounding tailored to the individual - their raw materials.).

CONCLUSION

1. The research of this final task is to obtain the control model of raw material inventory at an optimum for industry, which inventory model of Continuous

Review (Q, R) and Extend (Q, R) Inventory Model Under Lead Time and Ordering Cost reductions Backorder conditions.

2. Number of economic order (Q) and the inventory position must be booked at the optimal raw material (R), the number of possible stock out (B), the amount of safety stock (SS), and the optimal service level (USL).
3. Reduced cost of messages (S) and lead time (l) sending any materials that can reduce the total cost of supplies each year to Extend (Q, R) Inventory Model Under Lead Time and Ordering Cost reductions Backorder conditions.

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