PREDICTION OF STRUCTURAL RESPONSES USING NATURAL FREQUENCY MEASUREMENTS

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ABSTRACT

This paper demonstrates that prediction of structural responses can experience significant errors due to the modelling errors or uncertain input even for simple structures. The responses of two simple reinforced concrete beam, as as examples of simple structures, were measured to be compared with the prediction. To improve the predictions, the relationship between static stiffness and modal stiffness of a structure is determined, which provided a basis for using the measured natural frequencies replacing the calculated natural frequencies, where modelling errors are normally occur. Using the natural frequency measurements significantly improves the quality of the predictions of the two actual beams. As applications, to predict the static response of a real seven storey concrete building and the dynamic response of a simple supported reinforced beam subjected to jumping loads are developed. The prediction of static response of the building using the first two measured natural frequencies during construction is close to the numerical prediction, this is due to the building is a test building does not have secondary elements such as walls, beams and partitions in it. The quality of the prediction of the dynamic response of a concrete beam subjected to jumping loads improves significantly from the use of the measured natural frequency.

Keywords: frequency, measurements, prediction, structural responses, concrete, jumping loads

INTRODUCTION

Measurements which conducted on a real structure show the actual responses of the structure. However, prediction of structural responses can experience significant errors due to the modelling errors or uncertain input even for very simple structures. There is, therefore, a motivation to carry back analysis of the structure to compare with measurements with a view to improve guidance on the predictions (Reynold et al, 2005).

Impact test and static test were being performed on the two simply supported beams. Using the simple supported beam which provides the exact solution of responses is better than using the cantilever beam or more complicated supports (Wahyuni, 2007). The natural frequency as the basic structural dynamic characteristics was provided from the impact test, while the deflections of the beam are provided by the static test. The effect of boundary condition of the simply supported beam is investigated to reduce the error of the model. Any additional constrains, such as changing the pinned support to hinged support or fixed support, on the beam will increase the natural frequency and reduce the static displacement of the structure.

The stiffness of a structure is generally understood to be the ability of a structure to resist deformation. A structural stiffness describes the capacity of a structure to resist deformations induced by applied loads. If a discrete model of a structure is considered, the structural stiffness of a structure can be completely described by its stiffness matrix. However from its stiffness matrix, it may be difficult to be able to sense how stiff a structure is. In engineering practice a single value of the stiffness of a structure is preferred as it gives a direct indication of how stiff the structure is.

As the static stiffness and modal stiffness are defined independently and differently, the values calculated from the two definitions may be different for the same structure. However, as the two values are calculated on the basis of the same structure, i.e. using the same stiffness matrix, there should be a relationship between the static stiffness and modal stiffness of the structure. A relationship between static stiffness and modal stiffness of a structure has been provided by Wahyuni (2007), thus the relationship can be used to predict structural responses of structures.

The relationship between the static stiffness and modal stiffness provided a basis for using the measured natural frequencies replacing the calculated ones of the structure, where modelling errors are normally occur. The relationship of the two stiffnesses (Wahyuni, 2007) is

$$\frac{1}{K_s} = \sum_{i=1}^{r} \frac{\phi_{cl,i}^2}{K_{m,i}} \qquad (1)$$

where: K_s is the static stiffness, $K_{m,i}$ is the modal stiffness of the i th mode, $\phi_{cl,i}$ is the i th mode value.

This paper presents using the natural frequencies to predict the structural responses.

Firstly, the analytical and experimental analyses of the two simply supported reinforced concrete beams, which have been carried out to better understand how to the measured results can be used in prediction of the structural responses. Then applications are presented in the next section, which are to predict the response of a concrete building subject to static loading and to predict the dynamic response of a concrete beam subjected to jumping loads.

COMPARISON BETWEEN PREDICTIONS AND MESUREMENTS OF SIMPLY SUPPORTED BEAMS

The Tested Beams

Two reinforced concrete beams, one of the beams with dimensions of 0.4x0.08x3.0 metres in the University of Manchester (UM) as shown in Figure 1a, and the other with dimensions of 0.45x0.08x3.2 metres in the British Research Establishment (BRE) laboratory as shown in Figure 1b, are set up as simply supported beams for testing.

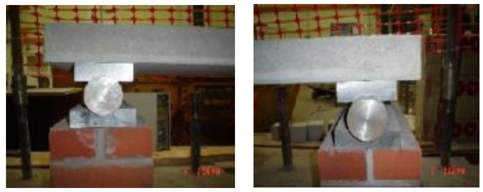
Simply supported conditions are normally used in calculations and experiments, but the realisation of such supporting conditions is unique. Therefore, two simply supported conditions are examined in the experiment with the beam is the same one. The first supporting condition is shown in Figure 2a, and the supports and the corresponding beam are called UM1 beam. The same beam with the slightly altered boundary condition (Figure 2b) is named UM2 beam. It can be seen from Figure 2b that the upper plates at the two supports are remove in the UM2 beam.

a. A test beam at UM

b. A test beam at BRE

Figure 1. The two test beams

The static tests and the impact tests were conducted on the two beams. The static test was conducted to measure the displacement of the beam subjected static loads and to determine the natural frequencies of the beams, the impact tests were carried out.



a. UM1 beam

b. UM2 beam

Figure 2. Boundary conditions of the UM beam

The dynamic loads, which are from jumping loads, were also conducted on the UM beams to obtain the dynamic response of the beam. Four individuals of known weight were instructed to jump at the centre of the beam with the frequency range from 2.0 Hz to 2.4 Hz. The results of this testing can be found in the section of jumping load.

Laboratory Test Procedure

Static tests

The static stiffness is defined by

$$K_{sm} = \frac{P}{\Delta_s} \qquad (2)$$

where P is the load applied at the centre of the beam and the displacement Δ_s at the centre of the beam is measured. Six sets of the test were conducted on the BRE beam while 22 sets were conducted on the UM beam. Each set of the test on the beam was based on the different applied loads. Using Picolog software to carry out data logging and analysis of the UM beam, while Impact software v 2.1 to analysis data of the BRE beam.

The procedure of the static test is described as follows:

- The displacement transducer is placed at the centre of the beam as shown in Figure 1b. The PicoLog data logger collected sets of measurements from the channels of an analogue to digital converter (ADC) and stored them in a disk (PC's store). The beam and the desktop PC used to record the data using PicoLog software in UM beam and Impact software in BRE beam.

- The desktop PC is then switched on to run the software. The program was set to record the data for 10 second. The position of the initial displacement shown on the screen monitor was recorded.
- A dead load is placed at the centre of the beam, and then the displacement-time history was recorded and saved as a PSD file. The testing procedure was repeated for the different weights of load.
- A person of known weight was invited to stand at the centre of the beam, which induced displacement of the beam. The same procedures as used for recording the data of the dead loads were carried out.

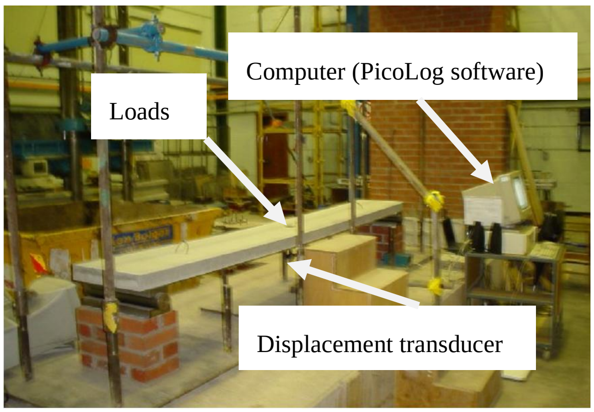


Figure 3. The beam and the instruments for static test

Impact tests

The fundamental natural frequency of a simple structure can be quickly and reasonably accurately obtained by an impact test. A DI-2203 FFT Structural Analyser fitted with an accelerometer (Figure 4) was used to carry out the frequency tests. A rubber hammer is used to knock at the centre of the beam once. The vibration of the beam generated by the impact is received by the accelerometer and sent to a FFT processor in the analyser.

The results produced in the time domain and frequency domain are displayed on the screen of the analyser. Peak values, shown on the frequency domain (Figure 5), correspond to natural frequencies of the beam. These data are saved in the analyser's internal memory and then transferred to a desktop PC.


Figure 4. DI-2203 FFT Structural Analyser

Procedures carried out for impact tests are summarised as follows:

- The accelerometer was mounted on a steel plate that was glued to the centre top of the beam. It was also connected to a BNC type interface on the structural analyser.
- A DC power supply on the structural analyser was turned on before applying the appropriate settings. The accelerometer input was set to 1V, bandwidth frequency to 500 Hz with the required time of 8 seconds. Frequency spectrum and time domain displays were chosen. A filename was entered to allow it to be saved in the analyser's internal memory. This data were later transferred to the desktop PC using a 9-25 pin serial cable.
- The start button was then pressed. A small tap using a soft rubber hammer was applied to the beam near the accelerometer to cause the beam to vibrate. Curves for both frequency and time domain shown on the screen monitor of the analyser are shown in Figure 5. The fundamental natural frequencies of the beam were identified by picking the peak values in the frequency domain. This step was repeated at least three times to ensure that the test and the results were repeatable and the average was taken as the fundamental natural frequency of the tested beam.

Using the frequency measurements, the modal stiffness of the fundamental mode can be calculated from:

$$K_m = M(2\pi\ f)^2 \qquad (3)$$

where K_m, M and f are the modal stiffness, the modal mass and the natural frequency of the fundamental mode of the system. The modal mass of the simply supported beam is calculated using $M = \overline{m}L/2$, where $\overline{m}$ is mass per unit length, and L is the length of the beam between the two supports.

Static stiffness and modal stiffness

In parallel with the measurement, analytical solution of the two beams can be conducted. The strain energy methods are used in the analysis as

The fundamental frequency of a simply supported beam is

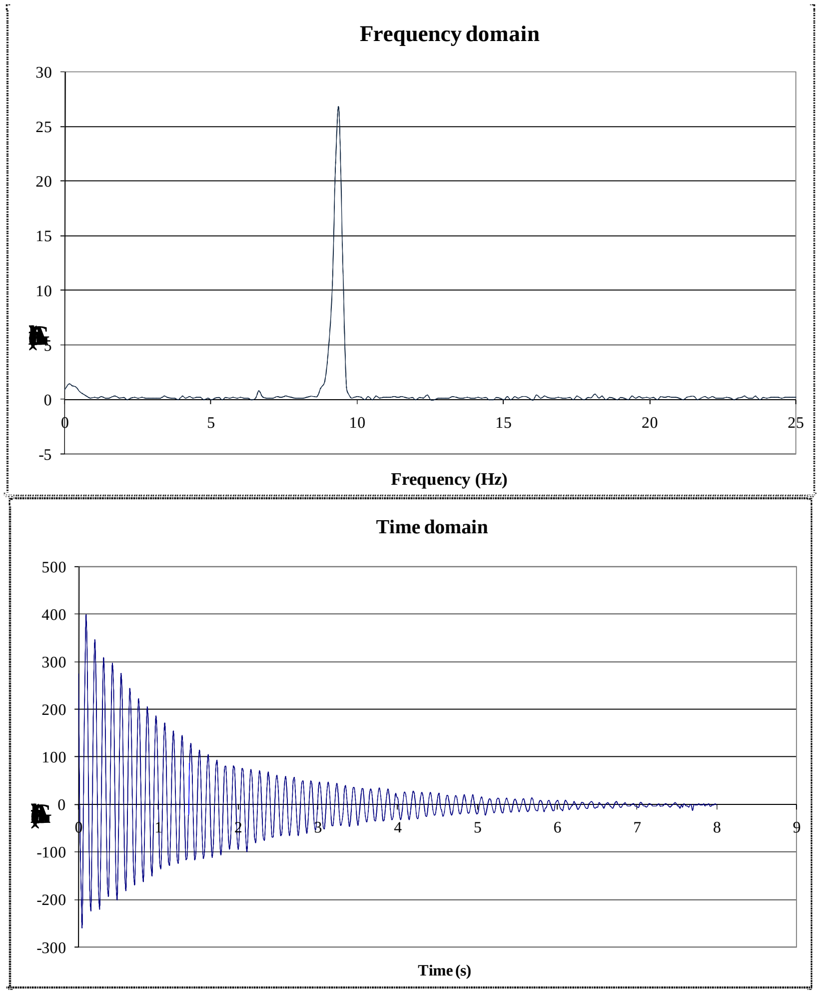


Figure 5. Peak values shown the natural frequency of the beam

Table 1. The comparison of the results from the measurements and calculations

Beams	Frequency, Hz			Modal Stiffness, N/m			Static Stiffness, N/m		
	Msrm	Predictions	%	Msrm	Predictions	%	Msrm	Predictions	%
UM1	9.625	14.44	150.0	4.708E+05	1.060E+06	225.2	4.324E+05	1.045E+06	150.1
UM2	9.375	14.44	154.0	4.467E+05	1.060E+06	237.3	4.232E+05	1.045E+06	154.1
BRE	17.76	14.84	83.6	1.406E+06	9.813E+05	69.8	1.434E+06	9.671E+05	83.55

*Msrm = measurements

$$f_{c,1} = \frac{\pi}{2L^2}\sqrt{\frac{EI}{m}} \qquad (4)$$

The modal stiffness of the beam is

$$K_{mc,1} = \frac{\pi^4 EI}{2L^3} \qquad (5)$$

The static stiffness of a structure is defined as the force applied to the structure, divided by the deflection it causes. When a concentrated load is applied at the centre of a simply supported beam, the derivation of the static stiffness can be stated as

$$K_{sc} = \frac{48EI}{L^3} \qquad (6)$$

where E is the Young's modulus of the beam is $30x10^9 N/m^2$ as the beam designed in the strength class C25, thus $E=9.5(f_{ck}+8)^{1/3}$ (BSI, 1997).

MEASURED AND CALCULATED COMPARISON

It is interesting to compare the measured and calculated results, and examine the differences between the two beams. The comparison all the results relating to measured and calculated values from the UM beam and BRE beam are summarised in Table 1.

The calculated natural frequency of the UM beam is 54% higher than the measured one. This corresponds to the calculated modal stiffness which is 137% higher than the measured one. The predicted static stiffness of the UM beam is 147% higher than the measurement. It demonstrates that the predictions of the UM beam are stiffer than it should be.

The predicted natural frequency of the BRE beam, shown in Table 1, is 16% lower than the measured one. This corresponds to the calculated modal stiffness, which is 30% less than the measurement. The calculated static stiffness of the BRE beam is 33% less than the measured one. It shows that the predictions of the BRE beam are less stiff than it should be. The differences are significant and thus it is important to identify the causes of the errors.

Identification of Errors

Table 1 shows the comparisons of the natural frequency and the static stiffness from the calculation and measurements. Two beams have been investigated, which one of the beam, the UM beam, is much less stiff than the predictions. The other beam, the BRE beam, is stiffer than the prediction. It is important to identify the errors.

Boundary conditions

There is a high probability that the supports can be a source of errors in the modelling. For this reason, two different supports were investigated to show the effect of the boundary condition as shown in Figures 2. The difference of the two supports is in a connection between the support and the beam, which the contact of the first support (UM1) is the connections in an area, and the second one (UM2) is in a line. The boundary condition may not be as ideal as a simple support. Decreasing in the contact area between the beam and the supports, the static stiffness of the beam decreases correspondently as given in Table 1.

Modulus of elasticity

The modulus of elasticity of the UM beam may be much smaller than the conventional value of $30x10^9$ N/m^2. This is one of the reasons why the measurements of the beam are smaller than the predictions. In contrary the BRE beam is stiffer than the prediction; the elasticity modulus of the BRE beam may be much bigger than the value of $30x10^9$ N/m^2 since the beam was constructed more than 10 years ago (Ellis and Ji, 1994).

Cracked beam

The existence of cracks in the UM beam contributes to decrease in the stiffness of the beam. Two cracked conditions of the beam are shown in Figure 6. It is not economical to use FE analysis to model this small detail as FE analysis is not well suited for this use unless special elements are used. If each stress raiser is surrounded by a profusion of small elements, meshing becomes tedious and computational demand becomes larger. These effects are not considered in analytical solutions.

Figure 6. Cracks in the UM beam

Using natural frequency measurements in calculations

Prediction of structural responses often leads to error due to inaccurate input or modelling. If the frequency measurement, which reflects the actual behaviour of a structure, can be used in calculation, it would remove some effects of inaccurate input or modelling.

For static analysis, the equilibrium equation is

$$[K]\{U\} = \{P\} \qquad (7)$$

For dynamic analysis or eigenvalue analysis

$$\{U\} = [\phi]\{z\} \qquad (8)$$

The displacement at the critical point in a particular direction is

$$u_c = \sum_{i=1}^{n \times d} \phi_{ic} z_i \qquad (9)$$

Substituting equation (8) into (7) and pre-multiplying $\{\phi_i\}$ lead to

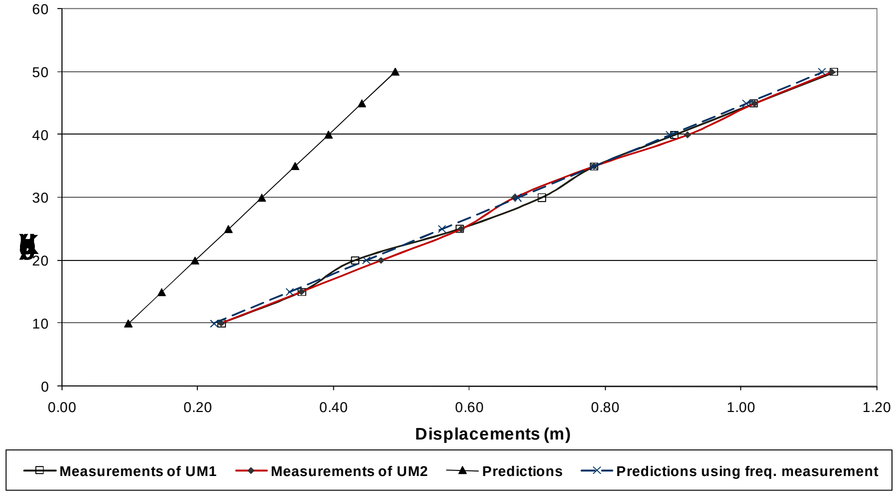


Figure 7. Static displacements of the UM beam

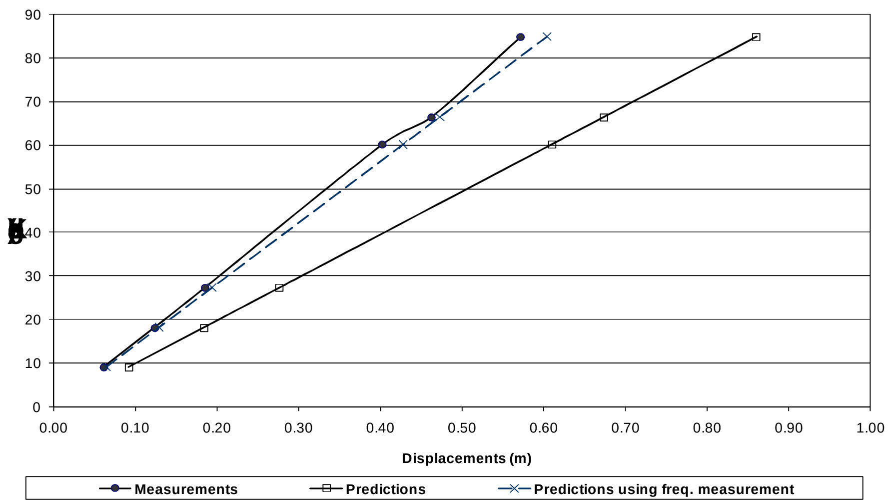


Figure 8. Static displacements of the BRE beam

$$\{\phi_i\}^T[K][\phi]\{z\} = \{\phi_i\}^T\{P\} \qquad (10)$$

i.e.

$$K_i z_i = \{\phi_i\}^T\{P\} \text{ or } z_i = \frac{\{\phi_i\}^T\{P\}}{K_i} = \frac{\{\phi_i\}^T\{P\}}{\omega_i^2 M_i} \qquad (11)$$

Where $\{\phi_i\}^T[K]\{\phi_j\} = \begin{cases} K_i & for \quad i = j \\ 0 & for \quad i \neq j \end{cases}$

Substituting equation (11) into equation (9) gives

$$u_c = \sum_{i=1}^{n\times d} \phi_{ic} \frac{\{\phi_i\}^T\{P\}}{\omega_i^2 M_i} \qquad (12)$$

There is no limitation on the load vector which can contain any real values. Equation (12) is an exact solution of the static displacement at the critical point, where ω_i can be either calculated or measured.

If there are only s measured natural frequencies available relating to u_c equation can be re-written as

$$u_c = \sum_{i=1}^{s} \phi_{ic} \frac{\{\phi_i\}^T\{P\}}{\omega_i^2 M_i} \qquad (13)$$

The advantages and disadvantages of using equation (13) to replace equation (7) are that

- It avoid to evaluation of the stiffness matrix, which often brings modelling errors.

- The measured natural frequency, ω_i, reflects the real behaviour of the structure to the stiffness and considers the contributions of all elements and connection conditions of the structure.
- As only first few natural frequencies can normally be measured, equation (13) provides as approximation of the response. If the static response is dominated by the first few modes, equation (13) will give a good estimation.

A good estimation given by equation (13) can be attributed to the fact that the inaccuracy due to eliminating the contribution from the higher modes is much less significant than the modelling error in the stiffness matrix in equation (7). The comparisons between the measured static displacement and the predicted ones are shown in Figures 7 and 8.

The ratios of measured displacements to predicted displacement range from 101% to 110% for the UM1 beam and from 99% to 105% for the UM2 beam and from 94% to 98% for the BRE beam. The differences between the measured and predicted displacements are about 10 microns. This indicates that the structural displacement subject to static loading can be predicted using measured natural frequency.

APLICATION

The applications of using natural frequency measurements to predict the structural responses are conducted on a concrete building and a simply supported beam.

Static Response of The Building

Using two first frequency measurements (Ellis and Bougard, 2001) is developed to predict the response of a concrete building subject to static loading. The building is a seven storey concrete building which designed using British standard (BSI, 1997). The building is modelled using software LUSAS (FEA, 2005), which already investigated in (Wahyuni and Ji, 2004).

Table 2. The comparison among the static displacements of the concrete building

Modes	Modal load $\{\phi_i\}\{P\}$	Natural frequency f, Hz	Modal Mass $M_{m,i}$, Kg	Mode shape at the critical point, $\phi_{cl,i}$	Modal displ. at the critical point due to each mode	Total modal displacement $u_{cl,k}$	Ratio,% $\frac{u_{cl,k}}{u_s}$	Ratio,% $\frac{u_{cl,k}}{u_{m,2}}$
1	2	3	4	5	6	7	8	9
Using natural frequency calculations, $u_{cl,k}$								
NS1	1.895E+04	0.619	1.120E+06	0.955	1.066E-03	1.066E-03	105.2	98.3
NS2	-7.969E+03	1.694	1.211E+06	0.982	-5.702E-05	1.009E-03	99.5	93.0
NS3	-5.702E+03	2.956	1.443E+06	-0.757	8.674E-06	1.018E-03	100.4	93.8
NS4	2.708E+03	4.208	1.116E+06	-0.621	-2.157E-06	1.016E-03	100.2	93.6
NS5	-2.828E+03	5.261	1.010E+06	-0.316	8.105E-07	1.016E-03	100.3	93.7
NS6	1.994E+03	6.446	1.217E+06	-0.128	-1.282E-07	1.016E-03	100.2	93.7
NS7	-1.359E+03	7.525	1.071E+06	-0.027	1.524E-08	1.016E-03	100.2	93.7
Using natural frequency measurements, $u_{m,i}$								
NS1	1.895E+04	0.600	1.120E+06	0.955	1.136E-03	1.136E-03	112.1	104.8
NS2	-7.969E+03	1.780	1.211E+06	0.982	-5.166E-05	1.085E-03	107.0	100.0
Static displacement calculated on static analysis, u_s						1.014E-03	100.0	93.5

The relationship between the static stiffness and the modal stiffness of the building was investigated in papers (Wahyuni, 2007). The calculated result indicated that the modal stiffness of the fundamental mode is larger than the static stiffness.

Five concentrated loads applied on the top of the building in the same direction, see Figure 9. The average displacement, u_s of the five-points is calculated. The static stiffness of the building from the static load can be worked out using equation (7), which is the displacement is the average of the five-points. The result is shown in the last row of Table 2.

The total modal displacements $u_{cl,k}$ of the building is determined by using equation (13) with the calculated measured frequencies as shown in the first row after title in Table 2. Using the first two measured natural frequencies of the building (Ellis & Bougard, 2001), the displacement of the building, $u_{m,i}$ is also predicted using equation (13). The comparison of the displacements using the calculated natural frequencies ($u_{cl,k}$), the measured natural frequencies ($u_{m,i}$) and the static load (u_s) is summarised in Table 2.

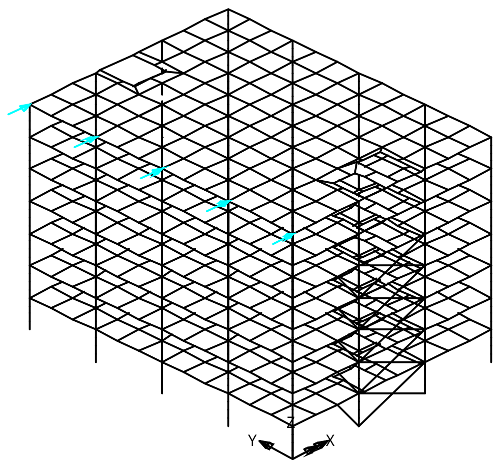


Figure 9. The five concentrated load at the top of the concrete building

It can be summarised from this study that:

- The modal displacement of the fundamental mode of the concrete building (1.066×10^{-3} m) is close to its static displacement (1.014×10^{-3} m). When the static analysis (the last row in Table 2) and equation (13) using the calculated natural frequencies are used to evaluate the static displacement of the building, equation (13) reaches 99.5% if the first two modes in the NS direction are considered and reaches 100.2% if the first seven modes are taken into account.
- When the first two natural frequency measurements and equation (13) are used, it gives 93.7% of the static displacement of the building. This is because the building, being a test building, does not have secondary elements such as walls, beams and partitions in it. In addition, the static displacement is calculated using equation (7) rather then the actual displacement.

The Dynamic Response of A Concrete Beam by Jumping Loads

Expression of a jumping load

The UM beam is also used to investigate the dynamic response induced by jumping loads. The load induced by an individual jumping can be expressed as summation of its self-weight and several harmonics as [3]:

$$F_s(t) = G_s\left(1.0 + \sum_{n=1}^{\infty} r_n \sin\left(\frac{2n\pi}{T_p}t + \phi_n\right)\right) \qquad (14)$$

where

$F_s(t)$ = time varying load
G_s = weight of the jumper
n = number of Fourier terms
r_n = Fourier coefficient (dynamic load factor)
T_p = period of the cyclic load
ϕ_n = phase lag of the n th term.

Equation (14) implies that the structural response can be calculated for each load component and the responses are later added up.

For jumping, the motion is defined by the ratio of the period that person is on the ground to the period of the jumping cycle, which is termed as the contact ratio, α, with the load during the contact period being represented by the half-sine wave. This can be used to determine the Fourier components of equation (15) shown as

$$r_n = \begin{cases} \dfrac{\pi}{2} & \text{for } 2n\alpha = 1 \\ \left|\dfrac{2\cos(n\pi\alpha)}{1-(2n\alpha)^2}\right| & \text{for } 2n\alpha \neq 1 \end{cases} \qquad (15)$$

and $\phi_n = 0$; *if* $2n\alpha = 1$

$$\phi_n = \begin{cases} \tan^{-1}\left(\dfrac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)}\right) - \pi & \text{if } \dfrac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} < 0 \\ -\dfrac{\pi}{\alpha} & \text{if } \sin(2n\pi\alpha) = 0 \\ \tan^{-1}\left(\dfrac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)}\right) & \text{if } \dfrac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} > 0 \end{cases} \quad \text{if } 2n\alpha \neq 1 \qquad (16)$$

For a SDOF system, with a natural frequency f and damping ζ, subjected to a harmonic load, with a frequency nf_p, the dynamic magnification factor M_n is

$$M_n = 1/\sqrt{\left(1-n^2\beta^2\right)^2 + \left(2n\zeta\beta\right)^2} \qquad (17)$$

where $\beta = f_p / f$

The response of a generalised SDOF system subjected to an individual jumping is then

$$A(t) = \Delta\left[1.0 + \sum_{n=1}^{\infty} M_n r_n \sin\left(\frac{2\pi n}{T_p}t - \theta_n + \phi_n\right)\right] \qquad (18)$$

with

$$\theta_n = \begin{bmatrix} \dfrac{\pi}{2} & \text{if } \left(1-n^2\beta^2\right) < 0 \\ \tan^{-1}\dfrac{2n\zeta\beta}{1-n^2\beta^2} & \text{if } \left(1-n^2\beta^2\right) > 0 \end{bmatrix} \qquad (19)$$

$$\Delta = \frac{G_s}{\omega_i^2 M_{m,i}} \qquad (20)$$

where ω_i = angular frequency in the i th mode
$M_{m,i}$ = modal mass of the structure

Equation (18) indicates that the dynamic displacement of a SDOF system induced by jumping loads is a summation of the static displacement and the dynamic displacement induced by harmonic

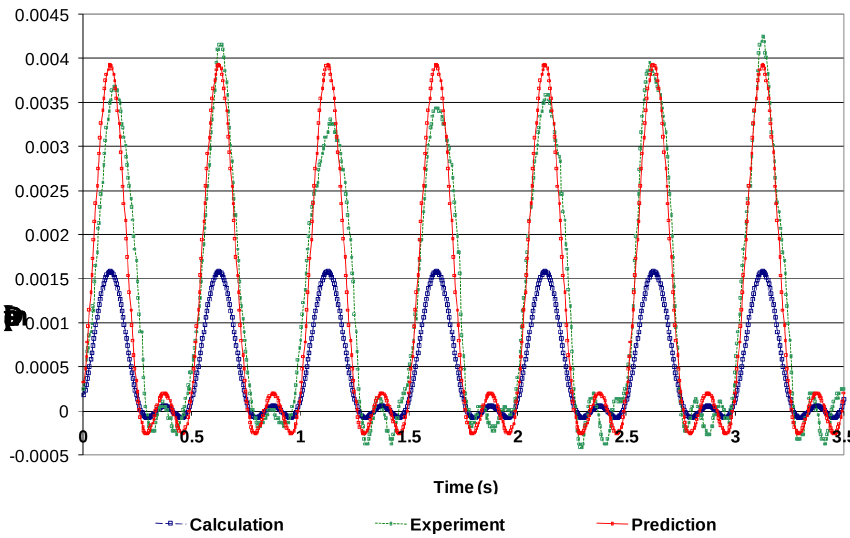


Figure 10. Dynamic response measurements, predictions and using frequency measurements in calculation of the simply supported beam induced by jumping loads of 2.0 Hz

functions at discrete loading frequencies, $f_p, 2f_p, 3f_p$, etc. From equations (15) and (16), Fourier coefficients and phase lags for different contact ratios are summarised in Table 3.

Table 3. Fourier coefficients and phase lags for different contact ratios (Ellis and Ji, 2004)

		n=1	n=2	n=3	n=4
α=2/3	r_n	1.28571	0.16364	0.13333	0.03643
	ϕ_n	-π/6	-π/6	-π/2	-π/6
α=1/2	r_n	1.5708	0.66667	0.00000	0.13333
	ϕ_n	0	-π/2	0	-π/2
α=1/3	r_n	1.80000	0.128571	0.66667	0.16364
	ϕ_n	-π/6	π/6	-π/2	π/6

Improvements to the prediction

The tests of the UM beam induced by jumping loads are carried out to obtain the dynamic displacements of the beam. The damping ratio of the simply supported concrete beam is assumed as 0.35% and a person who has a mass of 506 N was jumping at a frequency of 2.0 Hz. The contact ratio, α is ½ and three load components are considered in the prediction. Mathematica 5.1 software (Wolfram, 2002) is used to calculate the dynamic response subjected to the jumping load. The calculated natural frequency of the UM beam is 14.44 Hz. The predicted dynamic response of the simply supported beam is shown in Figure 10 (term: prediction).

To improve the predicted dynamic response of the beam, the calculated natural frequency in equation (20) is replaced by the measured natural frequency. The measured natural frequency of the UM beam is 9.625 Hz. The comparison between the dynamic responses from the measurement, prediction and prediction using natural frequency measurement are given in Figure 10.

As shown in the figure, the use of natural frequency measurement significantly improves the prediction of the dynamic response of the simply supported beam induced by jumping loads as indicated by the graph of the prediction using the natural frequency is close to the measurements.

Table 4 shows the further comparison between the maximum displacements of the simply supported beam obtained from:

- Dynamic displacement measurements of the beam to the jumping loads.
- Predictions using the calculated natural frequency in equation (20).
- Predictions using the measured natural frequency in equation (20).

It can be summarised from this study that:

1. The predicted dynamic displacements of the simply supported beam using the calculated natural frequency are too far away from those obtained by the measurements.
2. The quality of the prediction is improved significantly from the use of the measured natural frequency. This can be explained by the fact that the measured natural frequency includes the effect of Young's modulus, boundary condition, minor cracks, which cannot be modelled accurately in the analysis.

Table 4. Comparison between the dynamic responses of the simply supported beam induced by jumping load

Test No.	Person weight N	Load freq. Hz	Maximum displacements, mm				
			Measurements D_m	Predictions D_p	Ratio % D_p/D_m	Predictions f_m* $D_{p,fm}$	Ratio % $D_{p,fm}/D_m$
1	2	3	4	5	6	7	8
1	506	2.0	4.19E-03	1.587E-03	264.23	3.706E-03	88.36
2	506	2.2	4.35E-03	1.597E-03	272.22	3.762E-03	86.56
3	506	2.4	4.38E-03	1.607E-03	272.31	3.829E-03	87.49
4	527	2.0	4.31E-03	1.653E-03	260.47	3.859E-03	89.63
5	527	2.2	4.53E-03	1.663E-03	272.37	3.918E-03	86.51
6	527	2.4	4.58E-03	1.674E-03	273.41	3.988E-03	87.14
7	565	2.0	4.80E-03	1.772E-03	270.83	4.138E-03	86.20
8	565	2.2	4.16E-03	1.783E-03	233.47	4.201E-03	100.93
9	565	2.4	4.17E-03	1.794E-03	232.12	4.275E-03	102.64
10	612	2.0	4.92E-03	1.920E-03	256.33	4.482E-03	91.08
11	612	2.2	4.85E-03	1.931E-03	251.01	4.550E-03	93.88
12	612	2.4	4.94E-03	1.944E-03	254.22	4.631E-03	93.72
Average					259.92		91.85

* f_m is the measured natural frequency of the simply supported beam

CONCLUTIONS

The conclusion of this study can be summarised as follows:

1. The prediction of structural responses can experience significant error due to the modelling error or uncertain input even for very simply structures.
2. To improve the predictions, the relationship between static stiffness and modal stiffness of a structure is given, which provided a basis for using the measured natural frequencies replacing the calculated natural frequencies, where modelling errors are normally occur.
3. Using the frequency measurements significantly improves the quality of the predictions of the two actual beams and the concrete building.

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