MODELING INFLATION VOLATILITY USING ARIMAX-GARCH

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Abstract

Forecasting inflation is necessary as a basis for making decisions and high quality good planni ng in economic development in Indonesia particularly for the government and businessmen. T he forecasting generally uses time series data. However, there is a time series data which is dif ficult to obtain stationary, i.e., the variance on financial time series data such as the stock pric e index, interest rates, inflation, exchange rates, and etc. It is mainly caused by the inconsisten cy of variance (heteroscedasticity). This study developed Autoregressive Integrated Moving A verage (ARIMA) model using exogenous factors, namely the price of oil and outlier detection to forecast inflation. Another modeling which is expected to solve the problem of heterosceda sticity is a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. In th is study, the asymmetric GARCH of Glosten Jagannathan Runkle-GARCH (GJR-GARCH) w as carried out. This model could accommodate the volatility in the form of negative shocks th at can leverage the effect. The data used in this study was the Inflation rate of Indonesia and w orld oil prices in January 1991 to December 2014 respectively. The results showed that ARIM AX-GJR GARCH is the best model to forecast national inflation volatility.

Keywords: **inflation rate, ARIMAX, GARCH, GJR-GARCH**

Presenting Author's biography

Sri Aryani was born in Kendari, Sulawesi Tenggara province. She received the S.S.T degrees from STIS, Jakarta, in 2007. Since 2008, she has been government employee in Statistics Indonesia (BPS). She is currently a student of magister programme in department of statistics, Institut Teknologi Sepuluh Nopember, Surabaya.

1. Introduction

The economic life of a country could not be separated from macro-economic issues, among others are economic growth, inflation, unemployment, the stability of economic activity, and the balance of trade as well as balance of payments. According to [1], inflation is the general increase in prices of goods an d services which are basic needs for the community or the decline in purchasing power of the country's currency. In the case the inflation lifts up uncontrollable, the result in the value of money will be lower . High inflation is very important to note in view of the economic impacts that may cause economic ins tability, slow economic growth, and increased unemployment.

In general, inflation arises because of the pressure from the supply side (cost push inflation) and the de mand side (demand pull inflation). The increase of crude oil prices in the international market also lea ds to inflation augmentation. It will soon be followed by the rising of oil products prices, such as gasol ine and fuel oil [2]. Furthermore, because there is an attempt to substitute oil with other forms of energ y, the price of alternative energy sources will also raise up. A study on the impact of oil price fluctuati ons in the world for economic conditions in Indonesia using Vector Auto Regressive (VAR) was carrie d out [3]. The results showed that fluctuation of world oil prices indicates a positive effect on the r ate of inflation during the year. Meanwhile a similar study by [4] carried out in Pakistan also r eaffirmed similar positive impact of the fluctuation on inflation in Pakistan. This study aimed to modeling inflation using ARIMAX with asymmetric GARCH for optimizing model.

2. Literature Review

Heteroscedasticity problem can be solved by ARCH introduced by [5]. According to Engle, the application of ARCH method on time series data with heteroscedasticity setback is evidenced to have a significant role in improving the efficiency. By using this model, the variance time series data error is merely affected by the variables' error studied in the past. In 1986, Tim Bollerslev developed this model into a method called Generalized Autoregressive Conditional Heteroscedasticity (GARCH) [6]. The model is considered to provide a more concise and effective method that ARCH model does, since it can reduce the reliance on lag errors of the previous study.

In its development, GARCH is improved into several types of model. [7] states that it is unreasonable to use a linear function only for the residual variance. In many financial cases, asymmetric GARCH model have been evidenced to have better results than the symmetric GARCH model, as shown in [8] that models of inflation in several countries in Asia including Indonesia by comparing symmetric and asymmetric GARCH models.

One model of asymmetric GARCH is GJR-GARCH. This model is a development of GARCH that initiated by [9]. [10] studied the symmetric and asymmetric GARCH models in the time series of daily stock price the United States. In that study, Hentschel compared the method of GARCH, GJR-GARCH, TGARCH, AGARCH, NGARCH, EGARCH and APGARCH in forecasting. Furthermore, [11] conducted a study on the volatility of stock prices in Asia and Europe with some models of asymmetric GARCH like GJR-GARCH. Forecasting accuracy on every model is different depending on the country which is the domain of research. Furthermore [12] concluded from his research on forecasting the exchange rate index securities BRICS that the model of asymmetric GARCH (GJR-GARCH and EGARCH) is more accurate compared to Risk Neutral Historic Distribution (RNHD).

3. Methodology

The basic assumption that must be filled in the use of time series analysis is stationary data. Stationarity of variance can be detected from the value of the data that variance is constant. To cope with the heterogeneous variance data can be done by transformation. Transformation method often used is the transformation model introduced by Box and Cox in 1964. Stationarity in the mean would be achieved if the mean value of time series data being analyzed is not affected by time series. Data is determined to be stationary in the mean when it fluctuated around a line parallel to the axis of time (t) or around a constant mean value. The non-stationary data in the mean need process of differentiating (differencing) [13].

3.1 ARIMA and ARIMAX Model

Selection of an appropriate ARIMA model to a time series data can be done using the Box-Jenkins procedure. ARIMA model building procedure includes several stages of the identification, estimation, diagnostic checking, and forecasting. ARIMA (p, d, q) can generally be written as follows in Eq. (1) [14]:

$$\phi_p(B)(1-B)^d Z_t = \theta_0 + \theta_q(B)a_t \tag{1}$$

with:

θ_0 = constant

$\phi_p(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$

$\theta_q(B) = 1 - \theta_1(B) - \cdots - \theta_q B^q$

B = backward shift operator, $BZ_t = Z_{t-1}$

ARIMAX is an ARIMA model with exogenous factors. In this study, ARIMAX models that will be used is the outlier detection i.e. ARIMA model with exogenous factors dummy of outliers. It is also used ARIMA transfer function model.

To form the transfer function model , array of input and output of each series must have autocorrelation and a significant cross-correlation. Common forms of transfer function model to a single input (x_t) and single output (y_t) is shown in Eq. 2 [13]

$$y_t = v_0 x_t + v_1 x_{t-1} + \cdots + n_t \tag{2}$$

$$y_t = v(B)x_t + n_t$$

with y_t is representation of stationary output, x_t is representation of stationary input, and n_t is representation of error component (noise series) which follow an ARIMA model. $v(B) = v_0 B + v_1 B + v_2 B^k$ is a transfer function model coefficients or weighting impulse response, namely the composition of the weight of influence x_t to y_t in a dynamic system for the entire period of time that will come. The weight of the impulse response can be expressed as follows in Eq. (3) [13]:

$$v(B) = \frac{\omega(B)B^b}{\delta(B)} \tag{3}$$

then

$$y_t = \frac{\omega(B)B^b}{\delta(B)} x_{t-b} + \frac{\theta(B)}{\phi(B)} \alpha_t \tag{4}$$

with b is the number of periode before x_t began to affect y_t, $\omega(B) = \omega_0 - \omega_1(B) - \cdots - \omega_s(B)^s$ is the operator from order s, that represent the number of past observation of input x_t which affect output y_t. $\delta(B) = \delta_0 - \delta_1(B) - \cdots - \delta_r(B)^r$ is the operator from order r, that represent the number of past observation of output itself which affect output *yt.*

The steps of the formation of the transfer function model is identical to the steps in the formation of ARIMA models with illustrations as practiced by [13], namely:

Step 1: Identification

1. Prepare the input and output series

As in the case with ARIMA modelling, sequence of input and output series on modelling the transfer function requires stationary data. If the data is not stationary, it is necessary to establish differentiation or transformation.

2. Prewhiten the input series (x_t)

$$\phi_x(B)x_t = \theta_x(B)\alpha_t \tag{5}$$

Where α_t is white noise series with mean zero and variance σ_a^2. α_t series is as shown as:

$$\alpha_t = \frac{\phi_x(B)}{\theta_x(B)}x_t$$

3. Calculate the filter of output series (y_t)

That is transform the output series using the above prewhitening model to generate the series:

$$\beta_t = \frac{\phi_x(B)}{\theta_x(B)}y_t \tag{6}$$

4. Calculate the sample Cross-correlations Function (CCF), $\hat{\rho}_{\alpha\beta}(k)$ between α_t and β_t to estimate (v_k) as shown in Eq. (6):

$$\hat{v}_k = \frac{\hat{\sigma}_\beta}{\hat{\sigma}_\alpha}\hat{\rho}_{\alpha\beta}(k) \tag{7}$$

5. Determine the value of r, s, and b for the transfer function model.
6. Assessment of the initial series of disturbances (n_t) as follows in Eq. (7):

$$\hat{n}_t = y_t - \hat{v}(B)x_t \tag{8}$$

7. Determine (p_n, q_n) for ARIMA (p_n, 0, q_n) from noise series n_t.

Step 2: Estimating Parameter of Transfer Function Models

After identifying a tentative transfer function model, it needs to estimate the parameter:

$\boldsymbol{\delta} = (\delta_1, \dots, \delta_r)', \boldsymbol{\omega} = (\omega_0, \omega_1, \dots, \omega_s)', \boldsymbol{\phi} = (\phi_1, \dots, \phi_p)', \boldsymbol{\theta} = (\theta_1, \dots, \theta_q)'$, and σ_α^2.

Thus

$$\alpha_t = y_t - c_1 y_{t-1} - \cdots - c_{p+r}y_{t-p-r} - d_0 x_{t-b} + d_1 x_{t-b-1} + \cdots + d_{p+s}x_{t-b-p-s} + e_1\alpha_{t-1} + \cdots + e_{r+q}\alpha_{t-r-q}$$

where c_i, d_j, e_k are function of $\delta_i, \omega_j, \phi_k$, dan θ_l. The estimation method used is Conditional Maximum Likelihood, with assumption α_t is White Noise and normally distributed N(0, σ_a^2).

Step 3: Diagnostic checking of Transfer Function Models

1. Cross-correlation check between x_t and a_t

The following portmanteau test can be used as shown in Eq. (8)

$$Q_0 = m(m+2)\sum_{j=0}^{k}(m-j)^{-1}\hat{\rho}_{\alpha\hat{\alpha}}^2(j) \tag{9}$$

Q_0 follows χ^2 distribution with degrees of freedom (k+1)-m where $m = n - t_0 + 1$ and m the number of parameter δ_i and ω_j estimated in the transfer function $v(B) = \omega(B)/\delta(B)$.

2. Autocorrelation check

For an adequate model, both the sample ACF and PACF from $\hat{a}_t$ should not show any patterns. The following portmanteau test similar to Eq. (8) can be used as shown in Eq. (9)

$$Q_0 = m(m+2)\sum_{j=1}^{k}(m-j)^{-1}\hat{\rho}_{\hat{\alpha}}^2(j) \tag{10}$$

A time series data often contains observations that are influenced by extraordinary events which is not unexpected and unnoticed as a strike, the outbreak of war, political or economic crisis. It results these observations not consistent on time series data. Observations like these are called outliers [13]. Outliers can cause the data analysis becomes unreliable and invalid, so the outlier detection needs to be done to eliminate the effect of outliers. In this study, there are some observations contain outlier so it must be affect the model.

Outlier detection was first introduced by Fox (1972) in [13]. Outliers consists of several types, namely additive outlier (AO), innovational outlier (IO), level shift (LS) and the temporary change (TC). How to overcome the outlier is to include outliers in the model to get a model that meets the assumption of white noise and normal distribution.

3.2 ARCH/GARCH

To identify whether the model contains ARCH/GARCH, it can be done by calculating the value of ACF and PACF of squared residuals generated by the model mean (ARIMA) and can also use the test Lagrange Multiplier (LM). We chose the GARCH specification to model inflation volatility as there is much evidence available which suggest that the GARCH specification is better than ARCH. In an study about the performance of different volatility models, [15] find that while comparing the competing models on the basis of their out of sample predictive abilities, they do not have enough evidence to reject the hypothesis that none of other volatility models are better than GARCH(1,1).

Basically models with GARCH ARCH is equal. The difference is GARCH model not only depend on the squared error earlier time but also depends on the variance of an earlier time. While ARCH models only depend on the squared error earlier time. GARCH (p, q) expressed as in Eq. (10) [5]:

$\varepsilon_t \backslash \psi_{t-1} \sim N(0, h_t)$

with:

$$\begin{aligned} h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \beta_1 h_{t-1} + \beta_2 h_{t-2} + \cdots \\ &= \alpha_0 + \textstyle\sum_{i=1}^{p} \alpha_1 \varepsilon_{i-1}^2 \quad + \sum_{i=1}^{q} \beta_1 h_{i-1} \end{aligned} \tag{11}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1, \ldots, p$ and $\beta_i \geq 0$ for $i = 1, \ldots, p$.

GARCH is more parsimonious compared to ARCH as it captures the effect of infinite number of past squared residuals on current volatility with only three parameters and is less likely to breach non-negativity constraints artificially imposed on ARCH, [5]. But the primary restriction of GARCH model is that it enforces a symmetric response of volatility to positive and negative shocks. But the primary restriction of GARCH model is that it enforces a symmetric response of volatility to positive and negative shocks. According to [15], a positive inflation shock is more likely to increase inflation volatility via monetary policy mechanism, as compared to negative inflation shock of equal size. If this is true then we cannot rely on the estimates of symmetric ARCH and GARCH models and will have to go for asymmetric GARCH models. To capture those asymmetric responses of inflation volatility, we used asymmetric formulations of GARCH which is GJR model of [9].

3.3 GJR-GARCH

The GJR-GARCH, or just GJR, model of [9] allows the conditional variance to respond differently to the past negative and positive innovations. The GJR(1,1) model may be expressed as in Eq. (11):

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 \mathrm{I}(\varepsilon_{t-1} < 0) + \beta h_{t-1} \tag{11}$$

where I(.) denotes the indicator function. The model is also sometimes referred to as a Sign GARCH model. The GJR formulation is closely related to the Threshold GARCH, or TGARCH, model proposed independently by Zakoian (1994), and the Asymmetric GARCH, or AGARCH, model of Engle (1990) [17]. When estimating the GJR model with equity index returns, γ is typically found to be positive, so that the volatility increases proportionally more following negative than positive shocks. This asymmetry is sometimes referred to in the literature as a leverage effect, although it is now widely agreed that it has little to do with actual financial leverage.

3.4 Evaluation Criteria

Measuring tool that is used to calculate the prediction error is shown as :

a. Mean Square Error (MSE)

$$\mathrm{MSE} = \frac{1}{n}\sum_{t=1}^{n}(X_t - \widehat{X_t})^2 \tag{13}$$

b. Mean Absolute Error (MAE)

$$\mathrm{MAD} = \frac{1}{n}\sum_{t=1}^{n}\left|X_t - \widehat{X_t}\right| \tag{14}$$

c. Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{100\%}{n}\sum_{t=1}^{n}\left|\frac{X_t - \widehat{X_t}}{X_t}\right| \tag{15}$$

with

n = the number data

X_t = observation data in lag *t*

$\widehat{X_t}$ = forecasting results data in lag *t*

The smaller value generated by the three of measuring instrument, then the forecasting model used is better.

3.5 Data and Variable Set

The data used in this research is secondary data obtained from the Publication of Price of Statistics Indonesia (BPS). The data used is monthly inflation of Indonesia, ranging from January 1991 to December 2014. The number of data series used are 288 series. From January 1991 until December 2013 is used as an in-sample data or data training and data of January 2014 until December 2014 is used as the out-sample data or data testing. It also uses the percentage of change in world crude oil prices data ranging from January 1990 to December 2014 as an input series data.

Based on the background and purpose of the study, the research variables that will be used is the national inflation as a series output. Exogenous input variable used is variable the percentage of change in world crude oil prices. Furthermore, the detection of outliers is used in shaping the model ARIMAX.

4. Results

4.1 Results of ARIMAX and Symmetric GARCH

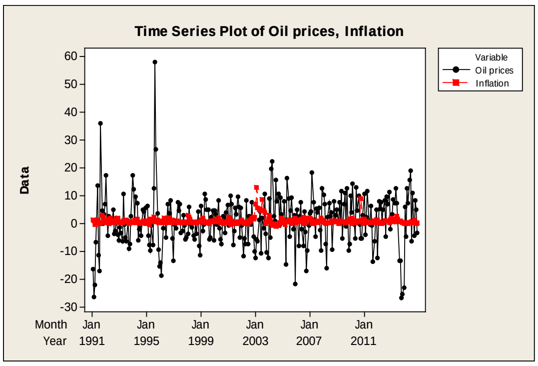

Fig. 1 Time series plot of oil prices and inflation

From fig. 1, we can observe that the pattern of oil prices is more volatile than inflation. This figure below shows the volatility of input series. The input series is stationary so the next step for building ARIMA tentative model can be done. From the results, the prediction of ARIMA is ARIMA(1,0,0) and ARIMA(0,0,1). Then the best model is ARIMA(1,0,0).

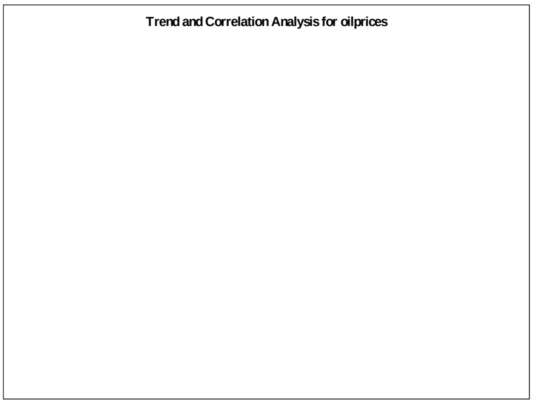

Fig. 2 Trend and Correlation Analysis for input series (World Oil Prices)

After ARIMA(1,0,0) is formed, the next step is to calculate the cross-correlation. The figure below shows that there is correlation between oil prices and inflation in the data series. Then, the tentative model of transfer function can be determined.

Fig. 3 Cross-correlation between input and output series

After forming the transfer function model, in fact, the residual of transfer function is not normally distributed. Then the next step is detecting the outliers. With outlier detection, it can combine with the transfer function model. The results of ARIMA transfer function with outlier detection shows that residual of ARIMAX model white noise and follows normal distribution.

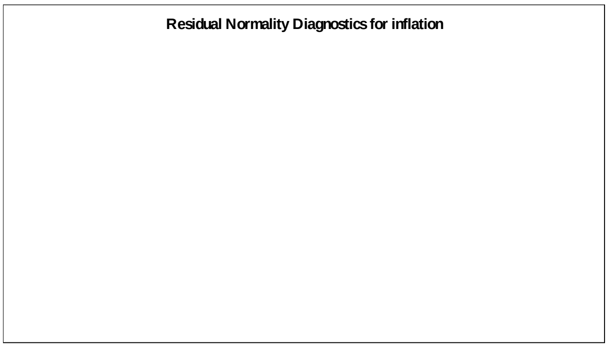

Fig. 3 Residual plot of normality

The results for the best model of ARIMAX with outlier detection is shown as:

$$Y_t = 0{,}715692 - (1 + 0.00478B)X_t + \frac{1}{(1-0.776B+0.09612B^8)} n_t + 1.724099I^4 - 1.76177\, I^{63} + 3.831641\, I^{85} + 8.524201\, I^{86} + 4.102888\, I^{91} - 2.38091\, I^{94} + 1.820892\, I^{97} + 1.892273\, I^{171} + 7.674444\, I^{178} + 2.214078\, I^{271}$$

The model shows that inflation in *t* month is affected by world oil prices in the previous month and ten additive outliers.

From the residual of ARIMAX model, it can be formed GARCH tentative model. This study checked the stability condition of two GARCH specification. There are some tentative models which can be used and tested respectively. The results of the final GARCH model is GARCH(1,1) shown as:

$$\sigma_t^2 = 0.147594 + 0.147120\, \varepsilon_{t-1}^2 + 0.425766\sigma_{t-1}^2$$

Under symmetric GARCH specification, all coefficient are positive. From the results above, we conclude that the variance residual of inflation is affected by quadratic residual in the previous month. That model can be determined as an ARCH model.

The asymmetric effect of GARCH can be seen on GJR-GARCH model below:

$$\sigma_t^2 = 0.094891 + 0.367113\, \varepsilon_{t-1}^2 + 0.579396\sigma_{t-1}^2 - 0.433146\, \varepsilon_{t-1}^2 I_{t-1}$$

From the results above, under asymmetric GARCH specification, all of coefficients are positive except the coefficient γ ($\gamma < 0$). It informs us the leverage effect of GARCH. It is expected and indicates the fact that negative inflation shocks in one period reduce the next period volatility. It means that

4.2 Performance Evaluation

Over the last decade or so, inflation forecasting has emerged as an important tool for regulators and financial consumers. The study wish to identify the optimal model of conditional variance, it seems appropriate to evaluate the out-of-sample forecasting performance of the variance equation. This paper evaluates the out-of-sample forecasting performance of GARCH models of conditional variance for national inflation. The specification of the conditional mean properly accommodates spikes, outliers, and secular movements of inflation. As for conditional volatility, this study consider two alternative classes of models – GARCH(1,1) and GJR-GARCH(1,1) – to accommodate inflation volatility. This study examined the monthly out-of-sample forecasting performance of the conditional variance models using three criteria (MSE, MAE, and MAPE). The results of the criteria of best models is shown in Tab.1:

Tab. 1 Out-of-sample forecasting results

Criteria	GARCH(1,1)	GJR-GARCH(1,1)
MSE	0.599580417	0.56755825
MAE	0.50525	0.492416667
MAPE	2.045653158	1.599112426

The MSE, MAE, and MAPE are measures of forecast accuracy that can be used to evaluate the performance of the conditional variance specification. Table 1 contains estimates of the MSE, MAE, and MAPE for all two volatility models over the out-of-sample forecasting horizon. The values between three criteria of two GARCH models respectively do not different significantly. Generally speaking, GJR-GARCH perform well in all criterias of best model. It is shown by the value of MSE, MAE, and MAPE of asymmetric GJR-GARCH that is less than symmetric GARCH.

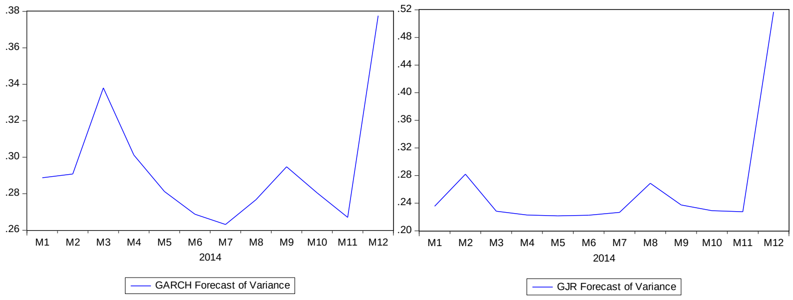

Fig. 4 Forecasting Plot of GARCH and GJR-GARCH

The results of out-of-sample forecasting obtained using the two GARCH models are listed in figure 4 that have a similar pattern. GARCH model tends to provide volatile forecasting compared to GJR-GARCH. Based on three criteria of best previous model, ARIMAX-GJR-GARCH model mostly yield better forecast results than other methods i.e. more precise prediction in national inflation data.

5 Conclusion and Suggestion

This study yielded several findings within the prevailing body of knowledge. It can be concluded that the asymmetric GJR-GARCH model performed better than symmetric GARCH in capturing inflation volatility in Indonesia. The increased volatility in inflation during the past year led to the urgency to find patterns that underlie this volatility. The accuracy of predictions indicated ARIMAX-GJR-GARCH is the most appropriate model to explain the inflation volatility.

The findings of this study may have implications on the way government/regulators and financial consumers anticipate the model of inflation volatility. Policymakers should be alert about the possibility of asymmetric model to extensively understand the importance of inflation stabilization program or the inflation targeting policies. It would help them in reducing the next period's volatility.

A number of improvements simulated in this study can be carried out, for instance, by inserting process ARIMA and a number of explanatory variables in the model. In addition, there are many variants of the ARCH models as well as FIGARCH model that can be used to predict the volatility. Finally, Markov regime-switching GARCH model was evidenced to have a great potential since it allows changes of the diverse parameters of GARCH volatility.

References

[1] Badan Pusat Statistik. *Metode Pengukuran Inflasi Indonesia*. Badan Pusat Statistik, 2008.

[2] A. Cologni, and M. Manera. Oil Prices, Inflation and Interest Rates in a Structural Cointregated VAR Model for G7 Countries. *Journal of Empirical Finance*, 30:83-106, 1993.

[3] M. A. Nizar. Dampak Fluktuasi Harga Minyak Dunia terhadap Perekonomian Indonesia, *Buletin Ilmiah Litbang Perdagangan*, page 189-209, 2012.

[4] S. Saleem, and K. Ahmad. Crude Oil Price and Inflation in Pakistan. *Bulletin of Business and Economics*, page 10-18, 2015.

[5] R. F. Engle. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Journal of Empirical Finance*, 50:987-1008, 1982.

[6] Bollerslev, Tim. Generalized Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics*, 31:307-327, 1986.

[7] Z. Ding, C.W.J. Granger, and R.F. Engle. A Long Memory Property of Stock Market Returns and A New Model. *Journal of Empirical Finance*, 1:83-106, 1993.

[8] S.K.A. Rizvi, et al. Inflation Volatility: An Asia Perspective, Economic Research. *Economic Research*, 27:280-303, 2012.

[9] Glosten, et al. On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. The *Journal of Finance*, 68:1779-1801, 1993.

[10] L. Hentschel. All in the Family Nesting Symmetric and Asymmetric GARCH Model. *Journal of Financial Economic*, 39:71-104, 1995.

[11] G. Forte, and M. Manera. Forecasting Volatility in Asian and European Stocks Markets with Asymmetric GARCH Models. *Newfin Working Paper at Bocconi University, Italy*.

[12] Labushagne, et al. A Comparison of Risk Neutral Historic Distribution E-GARCH and GJR-GARCH Model Generated Volatility Skews for BRICS Securities Exchange Indexes Analysis. *Procedia Economic and Finance*, page 344-352, 2015.

[13] Markidakis, et al. *Forecasting: Methods and Applications, 3rd edition*. John Wiley and Sons, 1998.

[14] W.W.S. Wei. *Time Series Analysis: Univariate and Multivariate Methods*. Addison-Wesley Publishing Co, 2006.

[15] A.D. Brunner, and G.D. Hess. Are Higher Levels of Inflation Less Predictable? A State Dependent Conditional Heteroscedasticity Approach. *Journal of Business & Economic Statistics*, 11:187-197, 2012.

[16] Suhartono and M.H. Lee. A Hybrid Approach based on Winter’s Model and Weighted Fuzzy Time Series for Forecasting Trend and Seasonal Data. *Journal of Mathematics and Statistics*, 7 (3), 177-183, 2011.

[17] Bollerslev, Tim. Glosary to ARCH (GARCH). *CREATES Research Paper*, 49, 2008.