ANTI-WINDUP CONTROLLER PARAMETERIZATIONS

Arief Syaichu-Rohman
School of Electrical Engineering & Informatics
Institut Teknologi Bandung, Jl. Ganesa 10, Bandung 40132, Indonesia
arief@stei.ee.itb.ac.id

ABSTRACT

Following a linear controller design, an anti-windup compensation is a popular approach that may be taken to deal with input saturation. There have been many anti-windup techniques proposed. Based on a transfer function parameterization of the resulting anti-windup controller, these anti-windup techniques may be classified into two categories, which may be called 1-degree of freedom (1-DOF) and 2-degree of freedom (2-DOF) parameterizations. Using newly known equivalence between a multivariable nonlinear algebraic loop and a constrained quadratic programming, two kind parameterizations of some existing anti-windup compensations are explained.

INTRODUCTION

Actuator saturation is a ubiquitous constraint that may induce adverse effects in any control systems. For linear systems, there have been many approaches proposed to overcome the effects of control input saturation. Anti-windup approaches (including conditioning techniques) are widely popular in which an anti-windup compensator is only active in the nonlinear region or a linear controller take control otherwise, see Fig.1.
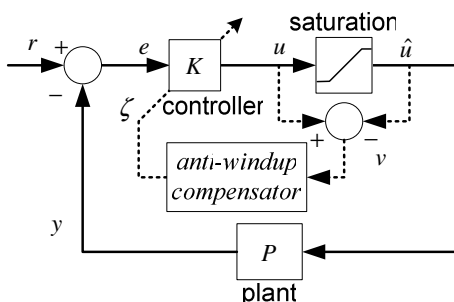

Fig.1 An anti-windup scheme

The anti-windup compensator may be a static gain matrix (e.g. (Mulder et.al., 2001)) or a dynamic transfer matrix (e.g. (Grimm et.al., 2003). Some of anti-windup schemes may include a nonlinear algebraic loop. It is illustrated in (Mulder et.al., 2001) that a nonlinear algebraic loop in an anti-windup scheme may improve the performance of its closed loop system under input saturation.

The resulting anti-windup controller is a combination of the nominal controller and anti-windup compensator. Based on a transfer function parameterization, these anti-windup approaches may be classified into two categories, which may be called 1-degree of freedom (1-DOF) and 2-degree of freedom (2-DOF) parameterizations. In fact, a unified view of some anti-windup approaches that has been presented in (Kothare et.al., 1994) may be presented in a 2-DOF parameterization. Meanwhile, the 1-DOF parameterization may be found in

(Edwards and Postlethwaite, 1998). However, a nonlinear algebraic structure that may arise in the anti-windup schemes has not taken into account properly in that parameterization.

Using the equivalence between a multivariable nonlinear algebraic loop and a constrained quadratic programming that has been recently known (Syaichu-Rohman et.al., 2003), two kind parameterizations, as in (Kothare et.al., 1994) and (Edwards and Postlethwaite, 1998) of some existing anti-windup compensations are revisited and then reformulated here.

Several static anti-windup schemes, i.e. the anti-windup compensator that has a zero order or a static matrix gain, are especially considered in this paper. The anti-windup (compensated) controller of the nominal one may be formulated as follows:

$$\dot{x}_c = A_c x_c + B_c e + \zeta_1 \qquad (1)$$
$$u = C_c x_c + D_c e + \zeta_1 = \bar{u} + \zeta_2 \quad (2)$$
$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} = -\Lambda v = -\begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix}(u - \hat{u}) \quad (3)$$

Note that the anti-windup compensation is only active when saturation occurs, i.e. $v \neq 0$.

NONLINEAR ALGEBRAIC LOOP

Consider a feedback system of Fig.2 in which Ψ represents a multivariable saturation function with ± 1 saturation level. It is proved in Syaichu-Rohman et.al, 2003) that a nonlinear algebraic loop in Fig.2 is equivalent to a constrained quadratic programming (QP) problem ($M=M^T>0$), that is
$$\hat{u} = \arg\min\left\{(u - \bar{u})^T M (u - \bar{u})\right\} (4)$$

subject to the constraint

$$-1 \leq u \leq 1. \qquad (5)$$
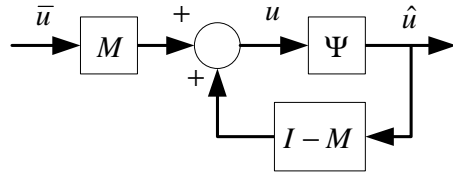
Fig.2 A multivariable nonlinear algebraic loop

Indeed, the algebraic loop in Fig.2 is a well-posed nonlinear algebraic loop and may be denoted as Ψ_M with the following relations:

$$u = M\bar{u} + (I - M)\hat{u}, \qquad (6)$$
$$\hat{u} = \Psi(u). \qquad (7)$$

In anti-windup schemes, a nonlinear algebraic loop may arise and is generally a well-posed non-symmetric loop, $M \neq M^T > 0$. Using (2) and (3), the following relation is obtained.
$$u = (I + \Lambda_2)^{-1}\bar{u} + (I - (I + \Lambda_2)^{-1})\hat{u}. \qquad (8)$$
Hence, it is clear that (7) and (8) form a multivariable nonlinear algebraic loop with

$$M := (I + \Lambda_2)^{-1}, \qquad (9)$$

and the loop will have a constrained QP equivalent problem if M is symmetric.

The parameterization of some anti-windup schemes (including some conditioning techniques) to regard the existence of the equivalent nonlinear algebraic loops within the schemes may now be formulated. The formulation will be put into both 1-DOF and 2-DOF using L and M as parameter matrices.

Referring to (Kothare et.al., 1994), the anti-windup approaches that are considered here are the one in (Mulder et.al., 2001) and conditioning techniques in (Hanus et.al., 1987), (Hanus & Kinnaert, 1989) and the multivariable version of (Walgama et.al., 1992). This version is a scheme with non-filtered *realizable reference*, see also (Peng et.al., 1998) that uses optimal *realizable reference* computation or optimal 'direction changer' as part of the anti-windup schemes. Analyzing those anti-windup schemes or approaches, the associated L and M parameter matrices may be obtained as the following.

In (Mulder et.al., 2001), the parameter matrices are

$$L := \Lambda_1 (I + \Lambda_2)^{-1} \text{ and } M := (I + \Lambda_2)^{-1}. \quad [10]$$

Meanwhile, the conditioning technique of (Hanus & Kinnaert, 1989) has

$$L := B_c D_c^{-1} \text{ and } M := D_c^{-T} D_c^{-1}. \quad [11]$$

Similarly, the scheme in (Peng et.al., 1998) uses the same L as in (Hanus & Kinnaert, 1989) but with

$$M := D_c^{-T} \Pi D_c^{-1}, \quad [12]$$

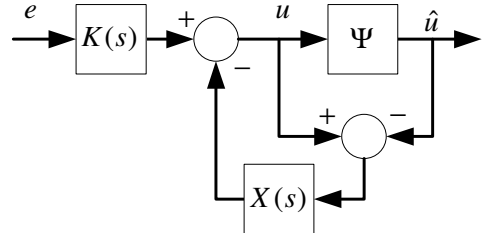for any user-chosen $\Pi = \Pi^T > 0$. As for the conditioning technique of (Walgama et.al., 1992), it has

$$L := B_c (D_c + \rho I)^{-1} \text{ and}$$
$$M := (D_c + \rho I)^{-T} (D_c + \rho I)^{-1}, \quad [13]$$

with $0 \le \rho < \infty$.

It is clear that defining $M = I$ means disregarding the algebraic loop or making $\Psi_M = \Psi$.

1-DOF PARAMETERIZATION

Consider a 1-DOF parameterization as illustrated by Fig.3 in which $K(s)$ is a nominal (linear) controller and $X(s)$ is an anti-windup compensator

Fig.3 1-DOF parameterization of anti-windup controller

In the 1-DOF parameterization, the anti-windup controller is parameterized by a linear transfer function $X(s)$ (see [Campo, Posthlethwaite]), *i.e.*,

$$u = (I + X(s))^{-1} K(s)e$$
$$+ (I + X(s))^{-1} X(s)\hat{u}. \quad [14]$$

The state space realization of the nominal controller $K(s)$ and the transfer function parameter $X(s)$ are as follows:

$$K(s) := \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}; X(s) := \begin{bmatrix} A_c & LM^{-1} \\ C_c & M^{-1} - I \end{bmatrix}.$$

Using the related L and M matrices to the anti-windup approaches, Table 1 presents the details of state space realization of $X(s)$. Note that if $M=I$ (without algebraic loop), then the parameterization results in (Edwards & Postlethwaite, 1998) will be obtained. Otherwise, the second column of the table is obtained.

2-DOF PARAMETERIZATION

Meanwhile, in the 2-DOF parameterization (see Fig.4), the controller is parameterized by two transfer functions $K_1(s)$ and

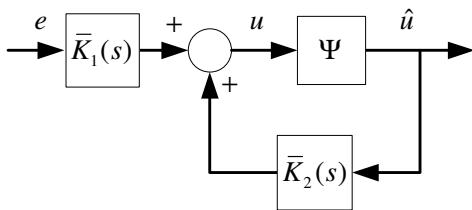$K_2(s)$ (see (Peng et.al., 1998)), or $\overline{K}_1(s)$ and $\overline{K}_2(s)$, *i.e.*,

$$u = \overline{K}_1(s)e + \overline{K}_2(s)\hat{u} := \overline{K}(s)\begin{bmatrix} e \\ \hat{u} \end{bmatrix}, \quad [15]$$

with the following state space realizations:

$$\overline{K}(s) := \begin{bmatrix} A_c - LC_c & B_c - LD_c & L \\ MC_c & MD_c & I - M \end{bmatrix}, \ [16]$$

where

$$\overline{K}_1(s) := \begin{bmatrix} A_c - LC_c & B_c - LD_c \\ MC_c & MD_c \end{bmatrix}, \text{and}$$

$$\overline{K}_2(s) := \begin{bmatrix} A_c - LC_c & L \\ MC_c & I - M \end{bmatrix}. \quad [17]$$

Fig.4 2-DOF Anti-windup Parameterization

Table 2 that is similar to Table 1 may then be devised for the case of 2-DOF parameterization. Again, if nonlinear algebraic loops are not to consider (i.e. *M=I*), some schemes based on conditioning technique will fit to the frameworks as reported in (Kothare et.al, 1994).

In addition, as presented in (Kothare et.al., 1994), the nominal controller may also be written as the following a left coprime parameterization :

$$K(s) := V(s)^{-1}U(s) = \overline{V}(s)^{-1}\overline{U}(s) \quad [18]$$

The associated 2-DOF parameterization transfer functions are

$$K_1(s) := U(s), \text{and } K_2(s) := I - V(s). \quad [19]$$

Now, if the nonlinear algebraic loop is to consider, the following transfer functions of the 2-DOF parameterization will be obtained.

$$\overline{K}_1(s) = MK_1(s) := \overline{U}(s),$$

$$\overline{K}_2(s) = I - M(I - K_2(s)) := I - \overline{V}(s) \ [20]$$

where $\overline{K}(s) = \begin{bmatrix} \overline{K}_1(s) & \overline{K}_2(s) \end{bmatrix}$.

It is then clear that matrix *M*, which is involved in the nonlinear algebraic loop, is the common factor of the left coprime parameterization of the nominal controller.

CONCLUDING REMARKS

Using newly known equivalence between a multivariable nonlinear algebraic loop and a constrained quadratic programming, two kinds of parameterizations of some existing anti-windup compensations are explained. In particular, the use of quadratic programming (QP) in some anti-windup schemes may now be modeled as a nonlinear algebraic loop that enables a more appropriate parameterization of the schemes.

Table 1 State space realization of $X(s)$

Anti-windup Scheme	$X(s)$	
	$M \neq I$	$M = I$
(Mulder et.al., 2001)	$\begin{bmatrix} A_c & \Lambda_1 \\ C_c & \Lambda_2 \end{bmatrix}$	$\begin{bmatrix} A_c & \Lambda_1 \\ C_c & 0 \end{bmatrix}^a$
Conditioning techniques	$\begin{bmatrix} A_c & B_c D_c^T \\ C_c & D_c D_c^T - I \end{bmatrix}^b$	$\begin{bmatrix} A_c & B_c D_c^{-1} \\ C_c & 0 \end{bmatrix}^c$
(Walgama et.al., 1992)	$\begin{bmatrix} A_c & B_c(D_c+\rho I)^T \\ C_c & (D_c+\rho I)(D_c+\rho I)^T - I \end{bmatrix}$	$\begin{bmatrix} A_c & B_c(D_c+\rho I)^{-1} \\ C_c & 0 \end{bmatrix}$

[a] $\Lambda_2 = 0$ (without algebraic loop)

[b] (Hanus, 1989) or (Peng et.al., 1998) with $\Pi = I$

[c] (Hanus, 1987)

Table 2 State space realization of $\overline{K}(s)$

Anti-windup Scheme	$\overline{K}(s) : M \neq I$
(Mulder et.al., 2001)	$\begin{bmatrix} A_c - \Lambda_1(I+\Lambda_2)^{-1}C_c & B_c - \Lambda_1(I+\Lambda_2)^{-1}D_c & \Lambda_1(I+\Lambda_2)^{-1} \\ (I+\Lambda_2)^{-1}C_c & (I+\Lambda_2)^{-1}D_c & \Lambda_2(I+\Lambda_2)^{-1} \end{bmatrix}$
(Hanus & Kinnaert, 1989) [a]	$\begin{bmatrix} A_c - B_c D_c^{-1}C_c & 0 & B_c D_c^{-1} \\ D_c^{-T}D_c^{-1}C_c & D_c^{-T} & 0 \end{bmatrix}$
(Walgama et.al., 1992)	$\begin{bmatrix} A_c - B_c(D_c+\rho I)^{-1}C_c & \rho B_c(D_c+\rho I)^{-1} & B_c(D_c+\rho I)^{-1} \\ (D_c+\rho I)^{-T}(D_c+\rho I)^{-1}C_c & (D_c+\rho I)^{-T}(D_c+\rho I)^{-1}D_c & 0 \end{bmatrix}$

Anti-windup Scheme	$\overline{K}(s) : M = I$
(Mulder et.al., 2001) [b]	$\begin{bmatrix} A_c - \Lambda_1 C_c & B_c - \Lambda_1 D_c & \Lambda_1 \\ C_c & D_c & 0 \end{bmatrix}$
(Hanus et.al., 1987)	$\begin{bmatrix} A_c - B_c D_c^{-1}C_c & 0 & B_c D_c^{-1} \\ C_c & D_c & 0 \end{bmatrix}$
(Walgama et.al., 1992)	$\begin{bmatrix} A_c - B_c(D_c+\rho I)^{-1}C_c & \rho B_c(D_c+\rho I)^{-1} & B_c(D_c+\rho I)^{-1} \\ C_c & D_c & 0 \end{bmatrix}$

[a] or (Peng et.al., 1998) with $\Pi = I$

[b] $\Lambda_2 = 0$ (without algebraic loop)

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