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Magnetohydrodynamic Computer Simulation of Erupting Low Solar Coronal Magnetic Fields

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Abstract: An analytical solution of initial and un-sheared low solar coronal magnetic arcade as initial value is perturbed and equilibrium is assumed to loss. The dynamical state of lossing equilibrium is descibed first and possible consequnces to the numerical scheme is derived. The dynamical evolution is followed by computer simulation which computed numerical solutions of complete set time-dependent magnetohydrodynamics partial differential equations. Automatic algorithm to inspect the MHD fast-mode speed is introduced to maintain simulation scheme stable during computations. Results show there will be shocks, known as MHD-waves shock, along the way to interplanetary space. Temperature shock is not prominent and only a relatively unsignificant temperature rise is resulted, instead density shock is noticeable. The density rise is relatively far greater to order of $1.0\times10^{9}kg$.

Key Words: Initial arcade, loss equilibrium, dynamical evolution-shocks.

1. INTRODUCTION

Since many scientific data (compiled from coronal-satellite for observing the solar corona: YOHKOH, TRACE, SOHO, and STEREO solar satellite missions [8]) become abundance, the solar corona looks much more active than previously thought. For example, in October 2003 the solar corona released the most powerful mass ejection that blinded-out navigation and wire-less radio communication for the entirely globe. It is of course might introduced biospheric temporal circumference changes in long terms [3].

In this occasion we wish to know from very beginning the solar coronal magnetic eruption just escaping from the solar surface by modelling an initial solar coronal model presumably pre-existing before eruption. In higher solar corona this phenomena is known as solar coronal mass ejection or solar CME. Solar CME terminology, abbreviated by CME, is frequently studied dynamically when it already attained some height of about 200,000 km above solar surface [2]. Rarely did it examine at much lower height.

Data from below 200,000 km is usually masked by occulting disk attached in front of objective lens of solar coronagraph telescope. It is why impossible to verify our results by comparing from data carried out from corona-graph [1]. Data from already mentioned above, for instance the YOHKOH data posed problems when we have to compare with dynamical case, because the data presented by the satellite are usually more static rather than dynamics. Further more the YOHKOH data expose the soft X-ray processes that it is more temperature data rather than plasma pressure data. Temperature generally effects from pressure evolution in low solar coronal magnetic arcade.

To task the problems and reach conclu-sive results, we implement analytical approach and numerical simulation. The analytical approach serves to seek plausible solution intended for solar coronal magnetic arcade structure prior to launch as a low CME. It will be in the domain of magnetohydrostatics (or MHS) analysis. Mean while the numerical approach serves as dynamical figures of about starting CME eruption in low solar corona. The dynamics figure of CME will be in the domain of magnetohydrodynamics (or MHD) physics.

Both the MHS and MHD analysis involves sophisticated mathematics and phy-sics because of non-linear interaction among plasma basic physical parameters. MHS analysis in even simplest form exhibit non-linear characteristics because the equation uses ‘nabla’ in magnetic and plasma pressure structures. While MHD analysis poses more complex structure of non-linearity and frozen-in plasma in magnetic fields. Numerical approach usually makes linear approximation for MHD analysis. As results numerical errors always appear and propagate during dynamical phase of simulated CME. An automatic algo-rithm to cure the errors and error propagations have to be innovated and included.

To attain conclusive results we implement the SHASTA (SHarp And Smooth Transport Algorithm) that had been used for solar-

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terrestrial problems [6][7]. This algorithm had been developed for institution which has no intensive and extensive computing facility. A desk-top or a lop-top with pretty fast processor is enough to do simulations.

2. MAGNETOHYDROSTATICS

Magnetohydrostatics or MHS is a special branch of plasma physics in high-temperature that integrates electron-plasma and magnetic fields into single state plasma physics. Magnetic fields treated as fluid-material, and does the electrons (and in some extent the protons) as well. All of those parameters are assumed in statical balances.

All assumed forces existed in initial magnetic structure before eruption as low CME; such as plasma viscousity, plasma pressure, magnetic force, and gravitational force are is statical balance. The circumference of physical situation is assumed pre-existing before perturbation. Other example see [9]. Equation for magnetohydrostatic balance is totally expressed as:

$$\nabla^2 {}^\circ V + \nabla p + (\nabla \times B) \times B + {}^\circ G = 0 \quad (1)$$

In the above equation, $^\circ$ is plasma density, p is plasma scalar pressure, B is magnetic fields, and G is gravitational vector. While V is velocity fields. The use of 'nabla' or ∇ is a sign that we are facing highly non-linear problems.

Expresion in first term is viscousity that proportional with momentum-density. The second term is plasma hydrostatic force. The third terms is Lorentz force due to inter-related of plasma and magnetic fields. And the last term is gravitational force acting to all plasma as bulge quantity.

Under very low plasma speed (<150 km/s) circumferences, the velocity vector is assumed equal to zero, or $V = 0$. Further more if the magnetic fields structure situated very low in the solar corona the gravitational strati-fication fulfiled $^\circ G = 0$. Situation under consideration makes equation (1) to become as a pure balance between plasme pressure gradient ∇p and magnetic Lorentz force $(\nabla \times B) \times B$, and relevan expression is

$$\boxed{\nabla p + (\nabla \times B) \times B = 0}. \quad (2)$$

Equation (2) may express in different appearence as below,

$$\frac{r}{dr}\left[p(r) + \frac{1}{2} B(r)^2 \right] + \frac{1}{r} B(r)^2 = 0 \quad (3)$$

which clearly a differential form for single parameter r as distance from center of un-sheared cylindrically symmetric magnetic arcade inhibit in low solar corona. Equation (3) implies that an integral form is possible as depicted below,

$$B^2 r^2 = -2\int r^2 \frac{dp}{dr}. \quad (4)$$

If we consider the particular pressure profile as below,

$$p = p_e + (p_0 - p_e)\exp(-r^2 / a^2), \quad (5)$$

posses a maximum value of $p(r) = p_0$ on the center of the axis $r = 0$. At large distance $r \to \infty$ the value falls to $p(r) \to p_e$ (inspect figure 1). Solution for magnetic fields structure has a magnetic topology as describe below,

$$B^2 = 2(p_0 - p_e)(a/r)^2 + -2(p_0 - p_e)[1 + (a/r)^2]\exp[-(r/a)^2] \quad \ldots\ldots(6)$$

It is obvious that magnetic structure describe a semi-circular cylindrical magnetic arcade for low solar corona. At far from the cylindrical axis the strength of magnetic fields decay to zero (inspect figure 2).

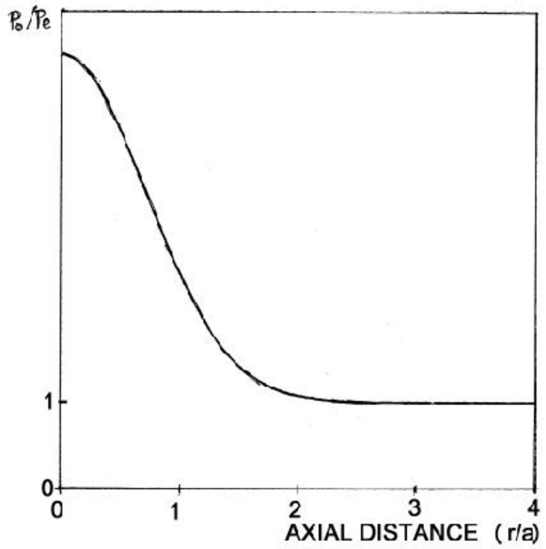

Figure 1: Graphical representation pressure profile as function of axial distance for initial low solar coronal magnetic arcade model which derived from MHS theory.

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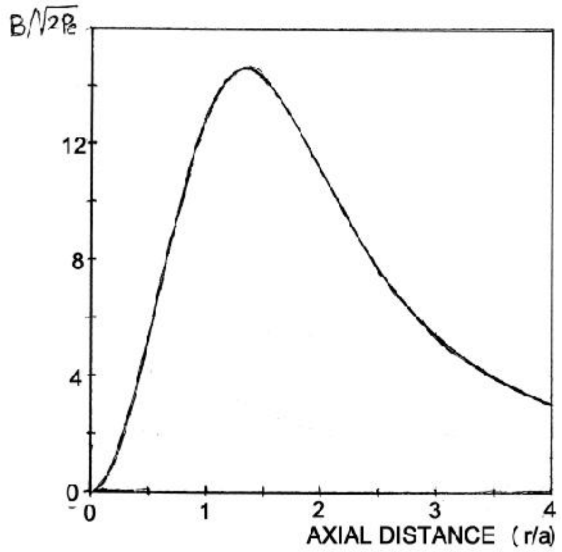

Figure 2: Magnetic field structure derived from assumed pressure profile. Magnetic field exhibits the semi-circular cylindrically symmetric before erupts.

3. MAGNETOHYDRODYNAMICS

Everything occurs on the solar surface are manifestation from static condition to dynamical situation. For examples, the solar flares, solar white-flares, solar prominence eruptions, including the CME are considered as lossing MHS equilibriums and evolved to dynamical states or MHD states.

Differences between MHS and MHD states are the MHS only solely concerns with space solutions, while the MHD states need both space and time solutions. The MHD needs to be handled with totally deferent approach. And even more, system that has developed to MHD states would introduce waves along the space and time evolution.

The MHD interactions need at least four basic plasma physics parameters to be included into ‘frozen-in’ concept of inter-related between electron cloud (in some extend with proton cloud) and magnetic fields.

They are plasma density °, momentum density $° V$, magnetic fields density B, and plasma pressure density p. Other forces such as gravitational force $° G$, solar differential momentum force $(\Omega \times V)$ are neglected because we consider low solar coronal magnetic aracade which will, in some reasons, erupt as low CME. We neglected the solar differential momentum because it is a long time scale process, while low CME is much shorter time scale.

A complete set of spatial and time dependent dynamical equations have to be utilized to cope the problems. The equations have to accomodate the privious concept. The complete sets are adopted as following,

$$\frac{\partial °}{\partial t} + \nabla.° V = 0 \quad (7)$$

$$\frac{\partial ° V}{\partial t} + \nabla.° VV = -\nabla p + (\nabla \times B) \times B \quad (8)$$

$$\frac{\partial B}{\partial t} + \nabla.BV = (B.\nabla)V + B(\nabla.V) \quad (9)$$

$$\frac{\partial p}{\partial t} + \nabla.pV = -(¥ - 1)\, p(\nabla.V) \quad (10)$$

The differential equations are implemented by time-to-time and point-to-point in space around solar coronal magnetic arcade prior to launch as low CME.

The most inportant to do this is an analytical detection of MHD waves. The waves are developed time-to-time and if it is not handled carefuly will hinder solutions. For the purpose we have to change the basic equations (7)-(10) by utilizing dynamical parameters. The waves nature may be included through special expressions to be integrated into basic sets differential equations (7)-(10). These parameters are depicted below,

$$\nabla \rightarrow +ik \quad (11)$$

$$\frac{\partial}{\partial t} \rightarrow -i\mu \quad (12)$$

following those equations (11) and (12), perturbation is applied to the system (7)-(10), such that mathematically may be written as

$$V \rightarrow V_0 + ¢V \quad (13)$$

$$° \rightarrow °_0 + ¢° \quad (14)$$

$$B \rightarrow B_0 + ¢B \quad (15)$$

$$p \rightarrow p_0 + ¢p \quad (16)$$

Substituted into equations (7)-(10), we have the following expressions,

$$-i\mu ¢° + ik.°_0 ¢V = 0 \quad (17)$$

$$-i\mu °_0 ¢V = -ik¢p + (ik \times ¢B) \times B_0 \quad (18)$$

$$-i\mu ¢B = ik \times (¢V \times B_0) \quad (19)$$

$$¢p = C_S^2\, ¢p \quad (20)$$

Equation (18) may be developed into an equation relates a special magnetic speed propagation, usually termed as Alfven speed (V_A^2) as follows,

$$-\mu^2 ¢V = -C_S^2(k.¢V)k + \\ -V_A^2\{k\times[k\times(¢V\times z)]\}\times z \quad (21)$$

Assuming the magnetic fields fullfil local topology as

$$B_0 \to B_0 z \quad (22)$$

then the Alfven speed is

$$V_A^2 = \frac{B_0^2}{°_0} \quad (23)$$

and the Hydrodynamic speed as

$$C_S^2 = ¥\,\frac{p_0}{°_0} \quad (24)$$

In MHD system then we have fast-mode speed (V_{FM}^2) as non-linear combination from equations (23) and (24),

$$\boxed{V_{FM}^2 = C_S^2 + V_A^2} \quad (25)$$

The fast-mode speed is unique since it appears only in MHD environment with special assumption of 'frozen-in' concept. The propagation pattern is also unique, which is a combination of hydrodynamics and magnetic propagation pattern, inspects figure (3) below.

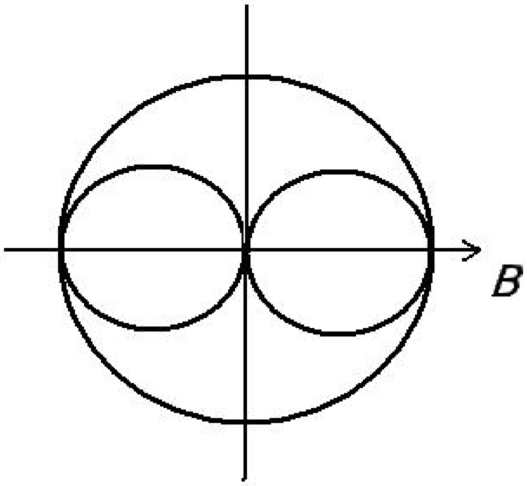

Figure 3: Propagation of magnetohydrodynamics waves as a combination of hydrodynamic speed and magnetodynamic (or Alfven) speed.

4. NUMERICAL SIMULATION

Numerical simulation is a relatively new scientific approach based on computer oriented algorithm as a benefit from ever increasing computing capabilities of certain CMOS processors. We have developed a com-puter algorithm with FORTRAN since the firstly capable desk-top 486-DX processor fast enough to run our numerical simulation algorithm. The maximum array can be handled, however, attained 81×81.

Heat resistence and faster processors such as PENTIUM III with LINUX operating system makes this work more challenging. Latest dual co-processors makes the algorithm works in a lap-top. Usually heat resistence lap-top is formidible to attain competitive results. Since our numerical simulation is a linear approximation of highly non-linear MHD processes an algorithm of controlling error with aproximate non-linear solution have to be automatically perfomed. Other wise erronous conclusions deviate our focus tasking a physical interactions on solar surface. At the same time clear, simple, and clean algorithm have to be innovated in such a way that expressions in syntax will always easy to inspect and correct mistakes.

For the shake of easiness representation, let the basic physical parameters $°, °V, B,$ and p , are symbolized to single quantity as Q . The quantity is convected numerically by a certain method such that numerical represen-tation is not far from original differential forms. For the purpose the sets of equations (7)-(10) may be re-written as a general form

$$\frac{\partial Q}{\partial t} + \nabla.QV = S_i \quad (26)$$

Where as Q is the quantity to be convected numerically, and S_i is various terms on the right side of equations (7)-(10). Our model is mathematically included in S_i where initial magnetic fields topology and initial plasma distribution in equations (5) and (6) are entered into the system under consideration.

For the purposes a little work has ro be done into Q as follows; firstly, define a numerical flux built from Q such as [6],

$$\left.\begin{aligned} F^0_{i+\frac{1}{2}} &= 2¦\,(Q^{t-1}_{i+1} - Q^t_i) \\ F^0_{i-\frac{1}{2}} &= 2¦\,(Q^t_i - Q^{t-1}_{i+1}) \end{aligned}\right\} \quad (27)$$

Advance Q using any finite difference expressions

$$\left.\begin{aligned} Q^{t+1}_{i+1} &= Q^{t-1}_{i+1} - \tfrac{\Delta t}{\Delta x}(Q^t_{i+2}V^t_{i+2} - Q^t_i V^t_i) \\ Q^{t+2}_i &= Q^t_i - \tfrac{\Delta t}{\Delta x}(Q^{t+1}_{i+1}V^{t+1}_{i+1} - Q^{t+1}_{i-1}V^{t+1}_{i-1}) \end{aligned}\right\} \quad (28)$$

Compute anti-diffussion in advance

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$$\left.\begin{aligned} F^{1}_{i+\frac{1}{2}} &= 2|\,(Q^{t+1}_{i+1} - Q^{i+2}_{i}) \\ F^{1}_{i-\frac{1}{2}} &= 2|\,(Q^{t+2}_{i} - Q^{t+1}_{i-1}) \end{aligned}\right\} \quad (29)$$

Apply the numerical diffusion in Q

$$\left.\begin{aligned} \overline{Q}^{t+1}_{i+1} &= Q^{t+1}_{i+1} + F^{0}_{i+\frac{1}{2}} - F^{0}_{i-\frac{1}{2}} \\ \overline{Q}^{t+2}_{i} &= Q^{t+2}_{i} + F^{0}_{i+\frac{1}{2}} - F^{0}_{i-\frac{1}{2}} \end{aligned}\right\} \quad (30)$$

To be second order precision take first difference of numerical diffused on $\overline{Q}$

$$\left.\begin{aligned} \Delta_{i+\frac{1}{2}} &= \overline{Q}^{t+1}_{i+1} - \overline{Q}^{t+2}_{i} \\ \Delta_{i-\frac{1}{2}} &= \overline{Q}^{t+2}_{i} - \overline{Q}^{t+1}_{i-1} \end{aligned}\right\} \quad (31)$$

Limits and correct by seeking negatives

$$\left.\begin{aligned} & S = sign(F^{1}_{i+\frac{1}{2}}) \\ & F^{C}_{i+\frac{1}{2}} = S.\max[0.\min(S.\Delta_{i-\frac{1}{2}}, \\ & \qquad |F^{1}_{i+\frac{1}{2}}|, S.\Delta_{i+\frac{1}{2}})] \end{aligned}\right\} \quad (32)$$

Apply numerical anti-diffusion by means of the corrected F^{C}

$$\left.\begin{aligned} Q^{t+2}_{i} &= \overline{Q}^{t+2}_{i} - F^{C}_{i+\frac{1}{2}} + F^{C}_{i-\frac{1}{2}} \\ Q^{t+1}_{i+1} &= \overline{Q}^{t+1}_{i+1} - F^{C}_{i+\frac{1}{2}} + F^{C}_{i-\frac{1}{2}} \end{aligned}\right\} \quad (33)$$

The MHD fast-mode has to be account for computational stabillity by controlling the computational time-step everytime new cycle by introducing relation as below

$$\Delta t \le 0.5 \Delta x / V_{FM} \quad (34)$$

The algorithm is then called 'fast-mode time-step control'. Every time new computational cycle automatically seeks greatest fast-mode that might develope during computations. The algorithm have to be automatically detected propagation that will across next grid.

The grid Δx is modified as well follow-ing equation (34). The grid is fixed but with a flexibility to follow fast-mode occurances,

$$\Delta x \rightarrow \Delta x + V_{FM}\ \Delta t \quad (35)$$

By that added equations (34) and (35), the algortihm is called as 'post-Eulerian' or 'pre-Lagrangin'. For low speed the algorithm approaching the 'fixed-grid' Eulerian concept, but for high-speed it automatically to be 'moving-grid' similar with Lagrangian con-cept. A 2D or 3D simulation will easily be derived form the above concept, see for eaxmple in [4].

Figure 4: Two dimensional simulation for low lying solar coronal magnetic arcade erupts as low solar coronal magnetic fields. Loop expose density and pressure enhancement, and in turn rise the temperature in the loop.

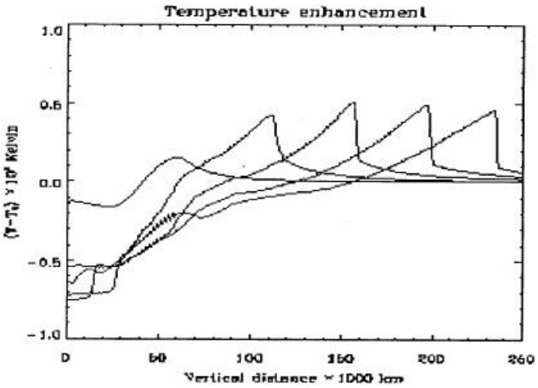

Figure 5: Temperature enhancement in moving loop of erupting magnetic arcade. Temperature is not directly convected paramater. Instead we compute pressure and density evolutions.

5. DISCUSSIONS

From MHS theory the possibility of getting unstable and erupt of cylindrical low solar coronal magnetic arcade could be happen by pressure or magnetic perturbations. The easiest way disturb the pressure profile by adding a little deviation form MHS pressure profile by proturding presure profile as equation (16). Then all the parameters will be perturbed consequently and build physical situation as describe by equations (13)-(16). If we wish to watch waves properties in the system then we have to introduce dynamical paramaters as pointed-out by transformations in equations (11)-(12). See other text in [5].

Dynamical situations have to be pre-scribed before creating numerical schemes. Especially the existance of the fast-mode MHD speed that might over-whelm the system. Carefuly define computational environment that controling the computational scheme in MHD perturbation is equally well with controling the MHD fast-mode.

Temperature rise only $0.5\times10^{6}K$ and it is meant that temperature only rises from $1.0\times10^{6}K$ to $1.5\times10^{6}K$. In term of solar corona it is relatively small number for coronal

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temperature rise. It is contrast with density rise in corona which attains $1.0\times10^{9}kg$ from initially only $1.0\times10^{2}kg$. Under assumption ideal gas relation $p = \circ T$ (non-degerated palsma gas), the pressure is not change ubruptly, instead the density. But since temperature is not directly convected by our numerical simulation, it is needed to develop other works to computed the temperature evolution in far more great details. If we wish to compute temperature it should be a little changes in numerical scheme by assign Q to directly convects temperature. Instead the pressure or densitiy have to be dropped not to directly convected.

A massive density waves originated from these CME will eventually changes the Earth orbital environment. Including perturbing solar-terrestrial communications and naviga-tions. Warning desimination for low CME is very important over the globe for long range aviations.

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