VIBRATION ANALYSIS OF DYNAMIC MODELLING OF GEAR PAIR IN MESH DUE TO TORQUE AND LOAD ON THE SYSTEM

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ABSTRACT

This paper presents dynamics modeling of gear pair in mesh. State space analysis and MATLAB operation are used to obtain natural frequencies and graphs of motion on the system. Torque load are applied to analysis numerical integration solution. The results are compared to the one without torque and load. The torque acts on pinion and the load acts on gear output. The natural frequencies of the gear pair in mesh system are calculated from the complex eigenvalues. The graphs of the gear angular velocity show that in star-up region, angular velocity increases with its rate velocity. It was vibrates with various amplitude. In steady state region, the gear angular has a constant average value. Applying the mesh stiffness variation affects in the changes of wave form and amplitude of the gear velocity vibration. The variable value of mesh stiffness causes vibration of the gear pair in mesh system. In addition, transmission error of the gear pair in mesh system tends to go into a constant value after periodically change. It is still vibrates in the gear system with the mesh stiffness variation.

Keywords: Dynamics modeling - gear pair – MATLAB - natural frequencies.

INTRODUCTION

Recent increase in using and operating of gear pair in the transmission system have led to intermittent noise and vibration problems in their gearing mechanism.

Dynamics modeling of gear pair in mesh is very useful to improve understanding of motion and vibration behavior of the system. The equation of motion can be arranged into the state space formulation base on vibration analysis and then with MATLAB operation supported by ODE solver (ODE23), graphs of motions and solution of problem of gear pair in mesh without damper can be derived. The solution provides data of the gear motion which can be plotted into the graph of body velocity or angular velocity of both pinion and gear motion.

To improve the current techniques of Gearbox vibration diagnosis and monitoring, many researchers are investigating the use of dynamic modeling of gearbox vibration to ascertain the effect of different types of gear train damage on the resultant gear case vibration (Randall, 1982). Modeling gear pair in mesh of this paper is based on the one developed by Du (1987) and subsequently modified by Howard et al. (2001).

Dynamic model of multiple pairs of gears in mesh including friction has been modeled and detailed by Howard et all (2001) and Further report was made by Rebbechi (1989) including friction and geometry errors.

PROBLEM STATEMENT

Developing model of gear pair in mesh is based on a single stage reduction gearbox where ratio of transmission is 1.75, and number of teeth for pinion is 36 and 63 for gear. In total, there are 16 degrees of freedom in the model and a schematic diagram as can be seen in figure 1.

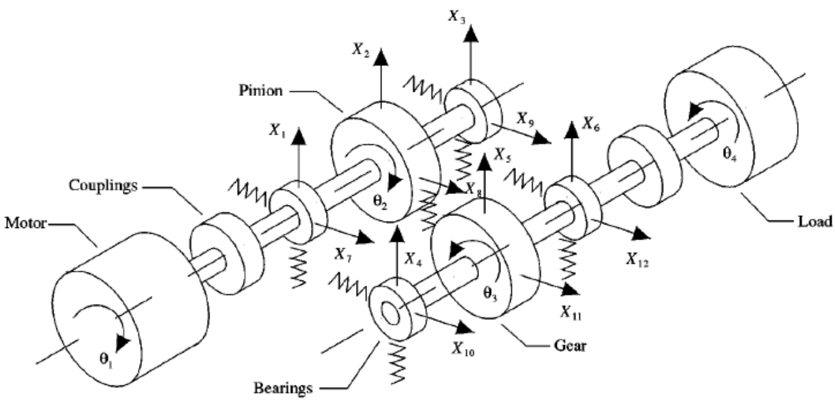

Figure 1: Diagram of the 16 degree of freedom gear dynamic model

Main assumptions are made as follows:

- Neglect of resonances of the gear case.
- Applying input torque and attaching load to the system.
- Shaft mass and inertia are lumped at the bearings or the gears.
- Neglect of shaft transverse resonances.
- Ignore of shaft torsional stiffness
- Damper that are occur in the system are neglected.
- Static transmission error effects are very much smaller than the dynamic transmission error effects and so can be neglected.

EQUATION OF MOTION

The simplified gear pair in mesh is shown in Figure 2 below.

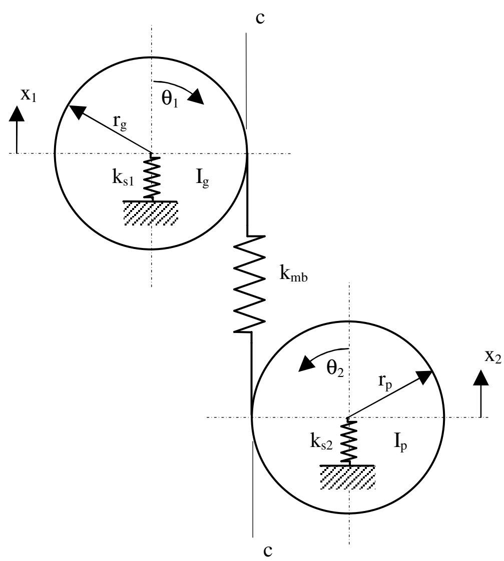

Figure 2: Coupling between the torsional and transverse motion in the gears and shafts

Where,
r_s : base circle radius of input pinion
r_g : base radius of output gear
k_s : shaft/bearing transverse stiffness
k_{mb} : linear translation tooth stiffness along line contact
c-c : line of contact.

The application of dynamic modeling to the vibration of simplified gear pair is reported in the following steps which are,

1. Derivation of the equation of motion for the four degree of freedom system.
2. The state space formulation
3. The solution for natural frequencies from the complex eigenvalues.
4. The solution of the motion of the gear pair after applying the input torque to the input pinion and attaching a load to the output gear.
5. The solution of the motion of the gear pair without damper.
6. Plotting the dynamic motion of gear body velocity and gear torsional velocity for start-up and steady speed.

From the figure 2, the equation of motion of gear pair in mesh can be derived as 4 degree of freedom of motion. 4 equations can be obtained from this case, as following:

obtained from this case, as following:

$$m_p\ddot{x}_1 + (k_{s1} + k_{mb})x_1 - k_{mb}r_p\theta_1 - k_{mb}x_2 + k_{mb}r_g\theta_2 = 0 \quad (1)$$

$$I_p\ddot{\theta}_1 - k_{mb}r_px_1 + k_{mb}r_p{}^2\theta_1 + k_{mb}r_px_2 - k_{mb}r_pr_g\theta_2 = 0 \quad (2)$$

$$m_g\ddot{x}_2 - k_{mb}x_1 + k_{mb}r_p\theta_1 + (k_{s2} + k_{mb})x_2 - k_{mb}r_g\theta_2 = 0 \quad (3)$$

$$I_g\ddot{\theta}_2 + k_{mb}r_gx_1 - k_{mb}r_pr_g\theta_1 - k_{mb}r_gx_2 + k_{mb}r_g{}^2\theta_2 = 0 \quad (4)$$

The matrix equation of motion can be written as

$$\begin{bmatrix} m_p & 0 & 0 & 0 \\ 0 & I_p & 0 & 0 \\ 0 & 0 & m_g & 0 \\ 0 & 0 & 0 & I_g \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} k_{s1}+k_{mb} & -k_{mb}r_p & -k_{mb} & k_{mb}r_g \\ -k_{mb}r_p & k_{mb}r_p{}^2 & k_{mb}r_p & -k_{mb}r_pr_g \\ -k_{mb} & k_{mb}r_p & k_{s2}+k_{mb} & -k_{mb}r_g \\ k_{mb}r_g & -k_{mb}r_pr_g & -k_{mb}r_g & k_{mb}r_g{}^2 \end{bmatrix} \begin{Bmatrix} x_1 \\ \theta_1 \\ x_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (5)$$

STATE SPACE FORMULATION FOR THE EQUATIONS OF MOTION

The matrix equation of motion will be solved by state space method, so the state space formulation should be constructed in order to obtain the natural frequencies and the solution of the motion of the gear pair.

$$\begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{\theta}_2 \end{Bmatrix} = \begin{bmatrix} -\frac{k_{s1}+k_{mb}}{m_p} & \frac{k_{mb}r_p}{m_p} & \frac{k_{mb}}{m_p} & -\frac{k_{mb}r_g}{m_p} \\ \frac{k_{mb}r_p}{I_p} & -\frac{k_{mb}r_p{}^2}{I_p} & -\frac{k_{mb}r_p}{I_p} & \frac{k_{mb}r_pr_g}{I_p} \\ \frac{k_{mb}}{m_g} & -\frac{k_{mb}r_p}{m_g} & -\frac{k_{s2}+k_{mb}}{m_g} & \frac{k_{mb}r_g}{m_g} \\ -\frac{k_{mb}r_g}{I_g} & \frac{k_{mb}r_pr_g}{I_g} & \frac{k_{mb}r_g}{I_g} & -\frac{k_{mb}r_g{}^2}{I_g} \end{bmatrix} \begin{Bmatrix} x_1 \\ \theta_1 \\ x_2 \\ \theta_2 \end{Bmatrix} \quad (6)$$

Let new variables of :

$$V_1 = x_1 \qquad V_3 = x_2$$
$$V_5 = \dot{V}_1 = \dot{x}_1 \qquad V_7 = \dot{V}_3 = \dot{x}_2$$
$$\dot{V}_5 = \ddot{x}_1 \qquad \dot{V}_7 = \ddot{x}_2$$

$$V_2 = \theta_1 \qquad V_4 = \theta_2$$
$$V_6 = \dot{V}_2 = \dot{\theta}_1 \qquad V_8 = \dot{V}_4 = \dot{\theta}_2$$
$$\dot{V}_6 = \ddot{\theta}_1 \qquad \dot{V}_8 = \ddot{\theta}_2$$

NATURAL FREQUENCIES OF THE SYSTEM

The natural frequencies of the system can be obtain by the

$$\{\dot{V}\} = [A]\{V\} \quad (7)$$

Let the assumption of:

$$\{V\} = \{Z\}e^{\lambda t}$$

$$\{\dot{V}\} = \lambda\{Z\}e^{\lambda t} \quad (8)$$

$$\lambda\{Z\}e^{\lambda t} = [A]\{Z\}e^{\lambda t}$$

$$\lambda\{Z\} = [A]\{Z\}$$

Note:

$\lambda\{Z\}=A\{Z\}$ is a standard eigenvalue and eigenvector problem.

$$[A - I\lambda]\{Z\} = 0$$

$$|A - I\lambda| = 0 \quad (9)$$

Applying initial condition and data for gear pair in mesh system below:

Table 1: Data for gear pair in mesh system

	Pinion	**Gear**
Gear type	Standard Involute	
Material	Steel 16MnCr	
Young's modulus (E)	205×10^9 N/m^2	
Density (ρ)	7850 kg/m^3	
Module	3.5	3.5
Number of teeth (z)	36	63
Gear ratio	1.75	
Pitch diameter (d)	12.881 cm	22.543 cm
Face width (b)	5.3 cm	5.3 cm
Contact ratio (c)	1.4	1.4
Pressure angle (α)	20°	20°
Gear mass (m)	5.422 kg	16.605 kg
Gear Inertia (I)	0.011246 kg m^2	0.105477 kg m^2
Bearing stiffness (k_s)	10^8 N/m	10^8 N/m
Mesh stiffness (k_{mb})	10^8 N/m	

The natural frequencies of the system can be obtained by substituting data in Table 1 above into the equation of $|A - I\lambda| = 0$, which gives 8 complex eigenvalues. 8 complex eigenvalues are:

$\lambda_1 = 0.0 + 0.0i$
$\lambda_2 = 0.0 - 0.0i$
$\lambda_3 = 0.0 + 692.2i$
$\lambda_4 = 0.0 - 692.2i$
$\lambda_5 = 0.0 + 2467.1i$
$\lambda_6 = 0.0 - 2467.1i$
$\lambda_7 = -0.0 + 4316.6i$
$\lambda_8 = -0.0 - 4316.6i$

The natural frequencies thus become

$\omega_i = [(Re(\lambda_i)^2 + Im(\lambda_i)^2]^{0.5}$

$\omega_1 = [(0^2 + 0^2)]^{0.5} = 0$ rad/sec
$\omega_2 = [(0^2 + 692.2^2)]^{0.5} = 692.2$ rad/sec
$\omega_3 = [(0^2 + 2467.1^2)]^{0.5} = 2467.1$ rad/sec
$\omega_4 = [(0^2 + 4316.6^2)]^{0.5} = 4316.6$ rad/sec

Or in the other shape of the natural frequencies in (Hertz) are

$f_1 = \omega_1 /2\pi = 0$ Hz
$f_2 = \omega_2 /2\pi = 110.17$ Hz
$f_3 = \omega_3 /2\pi = 392.65$ Hz
$f_4 = \omega_4 /2\pi = 687.01$ Hz

NUMERICAL SOLUTION OF THE GEAR PAIR WHEN AN TORQUE IS GIVEN AS AN INPUT TO THE INPUT PINION

Applying an input torque to the input pinion, makes the matrix equation of motion becomes

$$\begin{bmatrix} m_p & 0 & 0 & 0 \\ 0 & I_p & 0 & 0 \\ 0 & 0 & m_g & 0 \\ 0 & 0 & 0 & I_g \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} k_{s1}+k_{mb} & -k_{mb}r_p & -k_{mb} & k_{mb}r_g \\ -k_{mb}r_p & k_{mb}r_p^2 & k_{mb}r_p & -k_{mb}r_p r_g \\ -k_{mb} & k_{mb}r_p & k_{s2}+k_{mb} & -k_{mb}r_g \\ k_{mb}r_g & -k_{mb}r_p r_g & -k_{mb}r_g & k_{mb}r_g^2 \end{bmatrix} \begin{Bmatrix} x_1 \\ \theta_1 \\ x_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ T_1 \\ 0 \\ -T_2 \end{Bmatrix} \quad (11)$$

Torque data:
Torque input (T_{in}) : 75 Nm
Torque output (T_{out}): $0.0075\dot{\theta}_{gear}^2 = 0.0075\dot{\theta}_2^2$

The state space equation is solved by ODE Solver (ODE23) with MATLAB operation and applying initial conditions. The initial conditions are:
[V_1, V2, V3, V4, V5, V6, V7, V8] =
[0 0 0 0 0 0 0 0] and then time span is [0 t], solution of gear body velocity ($\dot{x}_2$) can be displayed in the plotted graph. Figure below present where time spans from 0 to 2 sec is.

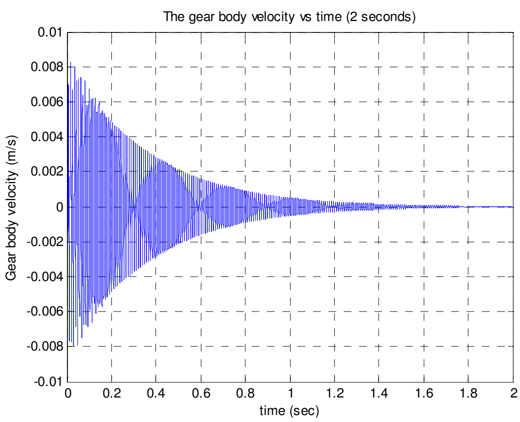

Figure 3: gear body velocity vs. time

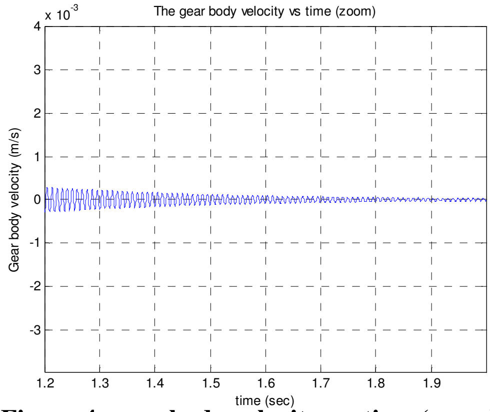

Figure 4: gear body velocity vs. time (zoom)

Moreover, solution of gear angular velocity ($\dot{\theta}_2$) can be displayed in the plotted graph in Figure 6 below, where is time spans from 0 to 2 sec.

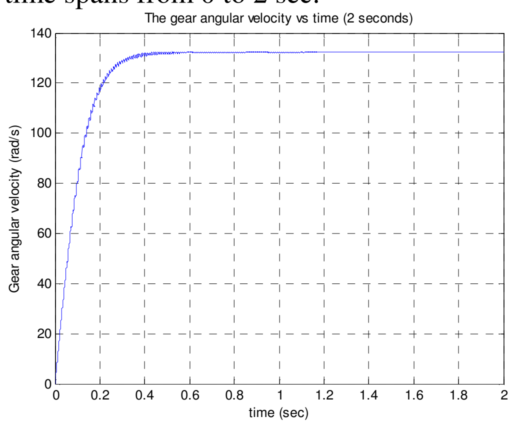

Figure 5: gear angular velocity vs. time

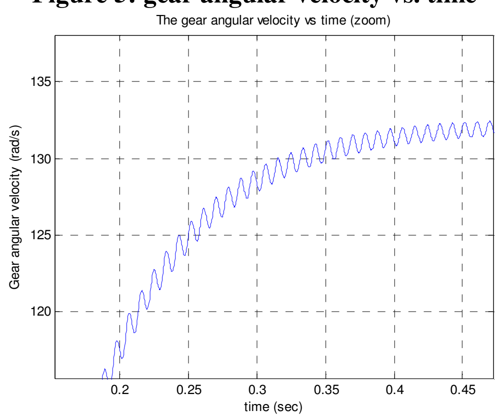

Figure 6: gear angular velocity vs. time (zoom)

Data that is produced by ODE solver can be used to display the transmission error of the gear system. **Figure 7 below** shows the transmission error between input pinion and output gear (θ_1-θ_2)

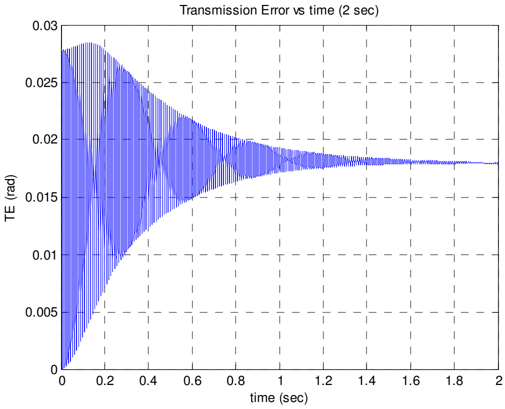

Figure 7 : Transmission Error of the gear system.

THE EFFECT OF MESH STIFFNESS VARIATION ON THE GEAR PAIR SYSTEM

Assuming that mess stiffness varies along the shaft rotation, which follows the equation of: (Lin & Parker, 2002, p.69)

$$kmb = ko + 2ka\sum_{i=1}^{\infty} a_i \sin(\Omega t) + b_i \cos(\Omega t)$$

and (12)

$$a_i = -\frac{2}{i\pi}\sin[i\pi(c-2p)]\sin(i\pi c)$$

$$b_i = -\frac{2}{i\pi}\cos[i\pi(c-2p)]\sin(i\pi c)$$

Where,

k_o : teeth stiffness (mean)
$2k_a$: peak-to-peak teeth stiffness
: (k_{max}-k_{min})
Ω : mesh frequency (number of teeth x angular velocity)
c : contact ratio
p : phasing angle

Mesh stiffness data
ko = 106 N/m
2ka = 6.105 N/m
Ω = z_1 x ω_1 = 36 x 231.5 =8834 rad/sec (=1326.4 Hz)
c = 1.4
p = 0

The 4 cycles of mesh stiffness with mesh frequency of 1326.4 Hz is shown in figure 8 below

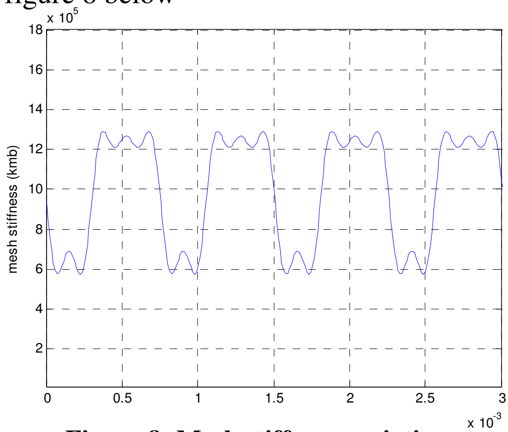

Figure 8: Mesh stiffness variation

DISCUSSION

Firstly, the natural frequencies of the gear pair in mesh system are found from the complex eigenvalues. When the damping isn’t included into the system, the complex eigenvalues only have imaginer components.

Secondly, the solution of dynamic motion which given by ODE solver shows that the angular velocity of the gear rises in two steps which are start-up and steady speed. In start-up region the gear angular velocity increases from 0 rad/s to 132.3 rad/s. During its increase the angular velocity vibrates with various amplitudes as can be seen in Figure 6 above. In addition, in steady state region, the gear angular velocity has a constant average value but still vibrates. The vibration amplitude tends to decay away, so the gear angular speed will reach a constant value of 132.3 rad/s. The interesting thing is in what frequency the angular velocity vibrates before it decays away. The question can be answer by check the period of the wave form. As can be seen in Figure 9, the period (T) is (0.506 sec – 0.497 sec) = 0.009 sec. Thus the frequency is (1/0.009) Hz ≅ 110 Hz which very close to the second natural frequency (=110.17 Hz) (the first natural frequency is 0 Hz).

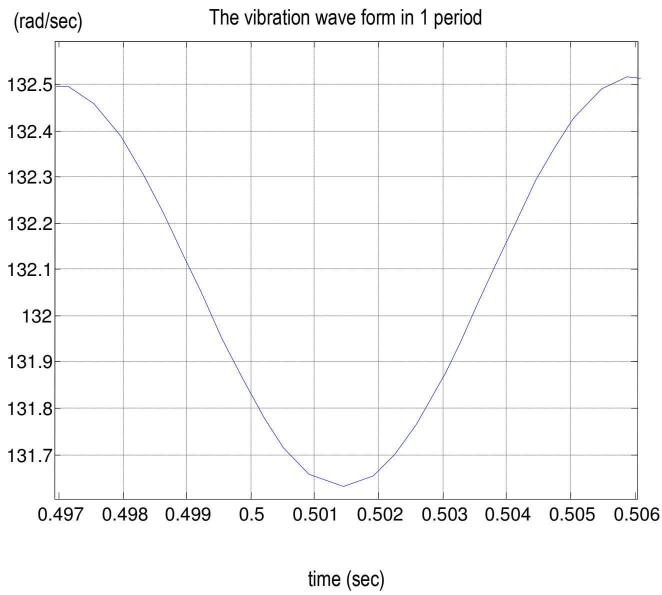

Figure 9: The angular velocity vibration wave form in 1 period.

Moreover, from figures above shows that although the damping is not included into the system, the vibration amplitude tends to decay away. It may possibly because the effect of applying the load in output gear. In the start-up region, the output torque increases proportionally to the square of gear angular velocity. The increasing of output torque also absorbs the energy from the system thus it acts as a damping.

Further, applying mesh stiffness variation into the gear pair in mesh system affects to the vibration amplitude, transient period time and frequency.

Finally, the solution also provides data that can be describe the transmission error between input pinion and output gear. Figure 10 shows the transmission error of the gear pair system before and after applying a damping and after uses the variable value of mesh stiffness. The transmission error in the two-first condition reveal that at start-up region the transmission error has a maximum value after that it decrease then tend to converge into constant value. However, in mesh stiffness variation condition, the transmission periodically changes in steady speed region.

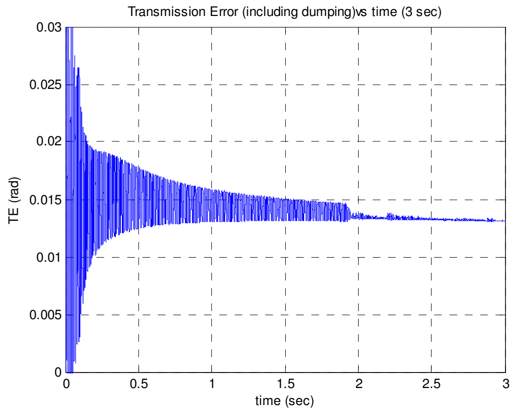

Figure 10: Transmission error of the gear system in variable mesh stiffness.

CONCLUSION

1. The natural frequencies of the gear pair in mesh system are found from the complex eigenvalues.
2. In start-up region the gear angular velocity increases into its rate velocity, it also vibrates with various amplitudes. Then in steady state region, the gear angular velocity has a constant average value but still vibrates. The vibration amplitude tends to decay away. Also, that vibration has a frequency which is very close to the second natural frequency.
3. Applying the mesh stiffness variation affects in the changes of wave form and amplitude of the gear velocity vibration. And the variable value of mesh stiffness causes vibration in the gear pair in mesh system.
4. Transmission error of the gear pair in mesh system tend to go into a constant value after periodically change is decay away. But, it still vibrates in the gear system with the mesh stiffness variation.

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